

Gravitational waves from phase transitions in the early Universe: sound waves and MHD turbulence

ETH/UZH Theoretical Particle Physics Seminar (Mar. 10, 2026)

ETH zürich



**University of
Zurich**^{UZH}

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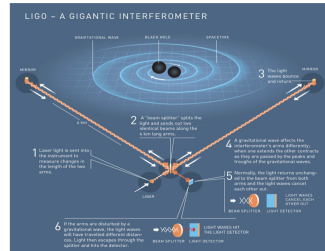
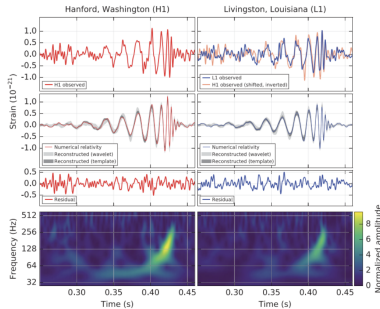


SNSF Ambizione grant (2023–2027): “*Exploring the early universe with gravitational waves and primordial magnetic fields.*”

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First detections of gravitational waves

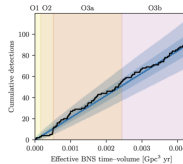
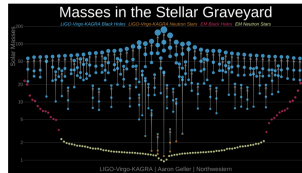
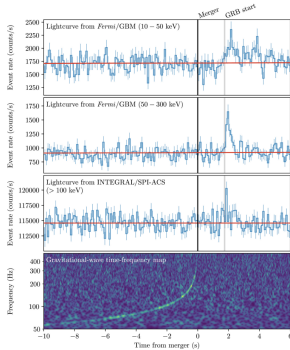
- Gravitational waves are opening a new window into our understanding of the Universe
 - First event GW150914 detected by LIGO-Virgo collaboration¹



¹[LIGO-Virgo Collaboration], *Phys. Rev. Lett.* **116**, 061102 (2016)

Introduction and Motivation

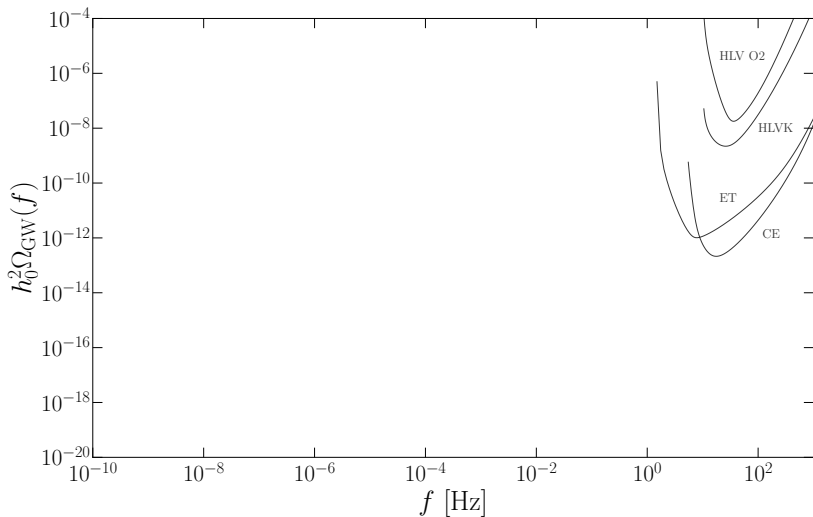
- GW170817 NS binary merger² first detection of GW and EM counterpart (constraint on the GW speed, measure of the Hubble rate, neutron star equation of state, ...)
- Several following events: LIGO-Virgo-KAGRA concluded the fourth observing run in November 2025 → ~ 340 events up to O4 (90 up to O3b)³ and 250 during O4.



²[LIGO-Virgo-Fermi GBM-Integral collaborations], Astrophys.J.Lett. 848 (2017) 2, L13

³[LIGO-Virgo Collaboration], GWTC-3, arXiv:2111.03606 (2021).

Gravitational spectrum (ground-based detectors)



LISA

- Laser Interferometer Space Antenna (LISA) is a space-based GW detector
- Approved in 2017 as one of the main research missions of ESA (L3) with NASA collaboration
- Launch planned for 2035
- Composed by three spacecrafts in a distance of 2.5M km
- LISA cosmology working group (since 2015, ~ 230 members)

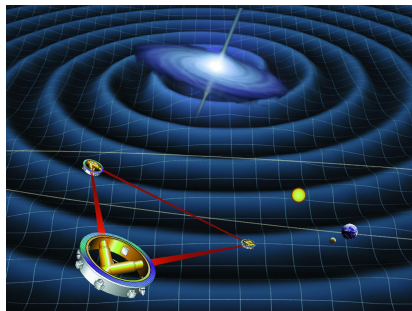
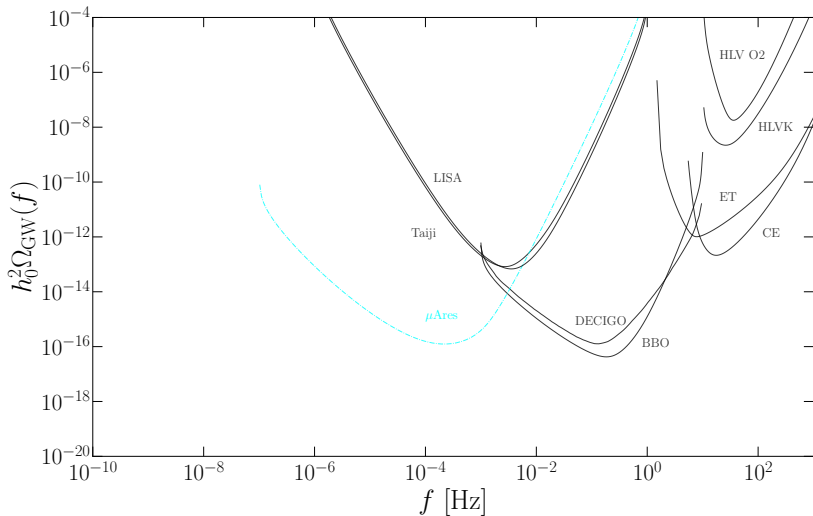


Figure: Artist's impression of LISA from Wikipedia

White paper:

[LISA Cosmology Working Group] (incl. ARP),
Living Rev. Rel. **26** (2023), arXiv:2204.05434.

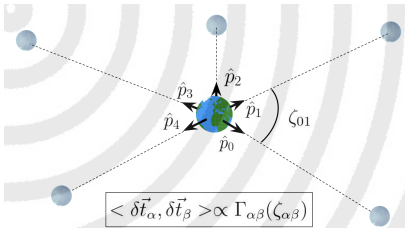
Gravitational spectrum (space-based detectors)



Pulsar Timing Array (PTA)

- An array of millisecond pulsars (MSP) is observed in the radio band to compute the delays on the time of arrival due to the presence of GWs.
- Collected data is the time series of residuals for each pulsar:

$$\delta t^i = t_{\text{obs}}^i - t_{\text{TM}}^i$$



Credit: Mikel Falxa

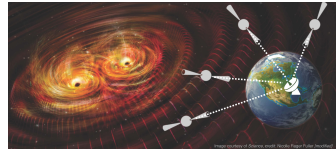
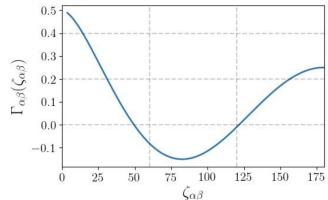


Figure: Image courtesy of Science, credit: Nicolle Rager Fuller

The correlation $\Gamma_{\alpha\beta}$ follows in GR the Hellings-Downs curve⁴

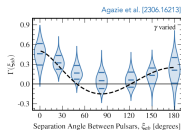


⁴R. W. Hellings and G. S. Downs, *Astrophys. J. Lett.* **265** (1983) L39-L42

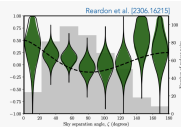
PTA detection

- The PTA collaborations reported for the first-time evidence of a stochastic gravitational wave background on a press release on June 28, 2023.

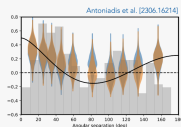
NANOGrav:
68 pulsars, 16yr of data
~3.4 σ significance



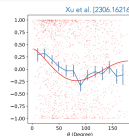
PPTA:
32 pulsars, 18yr of data
~2 σ significance



EPTA + InPTA:
25 pulsars, 24yr of data
~3 σ significance



CPTA:
57 pulsars, 3yr of data
~4.6 σ significance



Credit: Andrea Mitridate

Cosmological Gravitational Waves

- Gravity is the *weakest fundamental force*. Hence, GWs are difficult to detect but they propagate freely carrying *clean information of the source*.
- Cosmological GWs from the early Universe have the potential to provide us with *direct information on early universe physics* that is *not accessible via electromagnetic observations, complementary to collider experiments*.

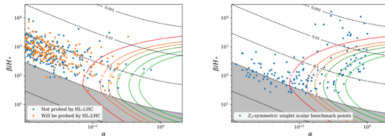
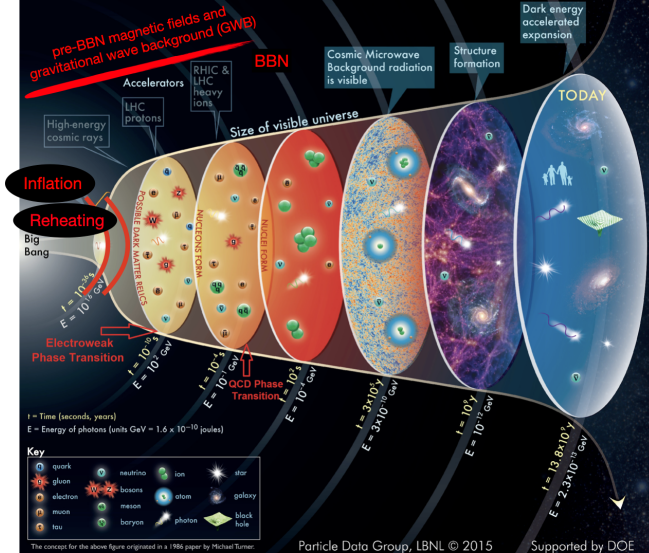


Figure 4: LEFT: Predicted values of α and β/H for $m_2 = 170$ GeV and 240 GeV (combined in one plot) in the general singlet model obtained by linearly varying the free parameters of the model and imposing requirements as described in the text. The mixing angles considered were $\sin \theta = 0.01$ (blue points) and 0.1 (orange points). The models in blue (orange) are unlikely (likely) to be probed by the high-luminosity LHC. The expected LISA sensitivities correspond to $T_* = 50$ GeV. RIGHT: Sensitivity curve for the Z_2 symmetric singlet extension. In both the left and right panels we have taken $v_w = 1$.

From [LISA CosWG], Caprini et al., *Detecting gravitational waves from cosmological phase transitions with LISA: an update (2019)*

pre-BBN magnetic fields and gravitational wave background (GWB)



The concept for the above figure originated in a 1986 paper by Michael Turner.

Particle Data Group, LBNL © 2015

Supported by DOE

Probing the early Universe with GWs

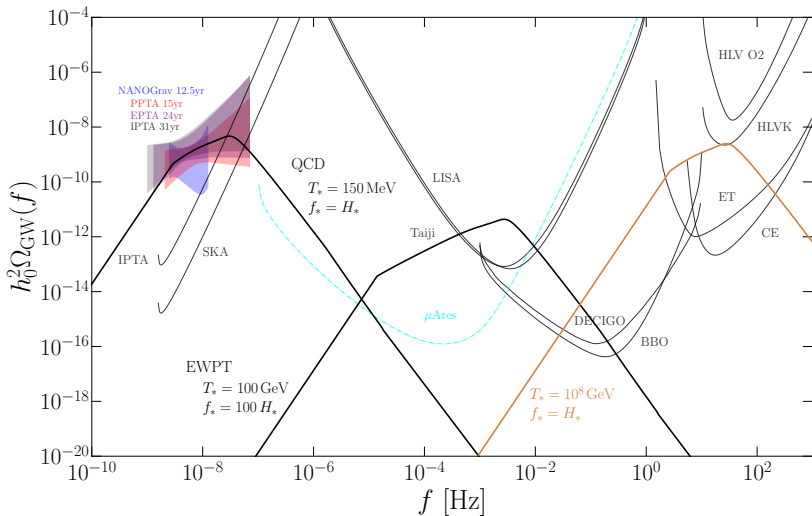
Cosmological (pre-recombination) GW background

- Why background? Individual sources are not resolvable, superposition of single events occurring in the whole Universe.

$$f_* \simeq 1.64 \times 10^{-3} \frac{0.01}{R_* \mathcal{H}_*} \frac{T_*}{100 \text{ GeV}} \text{ Hz}$$

- Phase transitions
 - Ground-based detectors (LVK, ET, CE) frequencies are 10–1000 Hz
Peccei-Quinn, B-L, left-right symmetries $\sim 10^7, 10^8$ GeV.
 - Space-based detectors (**LISA**) frequencies are 10^{-5} – 10^{-2} Hz
Electroweak phase transition ~ 100 GeV
 - Pulsar Timing Array (**PTA**) frequencies are 10^{-9} – 10^{-7} Hz
Quark confinement (QCD) phase transition ~ 100 MeV

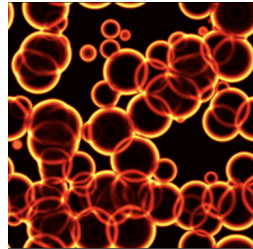
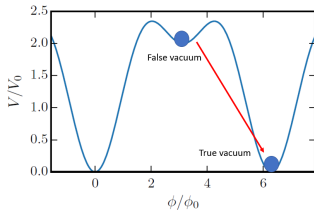
Gravitational spectrum from PTs⁵



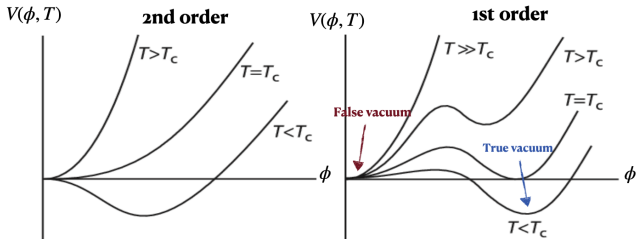
⁵ ARP, C. Caprini, A. Neronov, D. Semikoz, *PRD* **105**, 123502 (2022)
A. Neronov, ARP, C. Caprini, D. Semikoz, *PRD* **103**, L041302 (2021)
ARP *et al.*, arXiv:2307.10744 (2023).

First-order phase transition

$$V(\phi, T) = \frac{1}{2}M^2(T)\phi^2 - \frac{1}{3}\delta(T)\phi^3 + \frac{1}{4}\lambda\phi^4$$



Credits: I. Stomberg

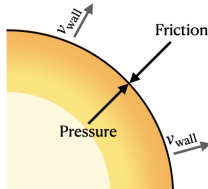


First-order electroweak phase transition

- The Standard Model (SM) predicts a crossover for the electroweak phase transition for the measured mass of the Higgs boson, 125 GeV.
- SM extensions: singlet or triplet scalar extensions, two Higgs doublet, Higgs portals, composite Higgs, scale invariant scalar extensions, SUSY models, low scale higher dimension operators.
- BSM additions are motivated by electroweak baryogenesis (Sakharov conditions, 1967).
 - ① C and CP violation: particles scattering off the bubble walls, producing asymmetries.
 - ② Baryon number violation: sphaleron transitions in front of the wall are biased to produce more baryons than antibaryons.
 - ③ Out of equilibrium: the bubble walls disturb the symmetric-phase equilibrium state.

Hydrodynamics of first-order phase transitions⁶

- Broken-phase bubbles are nucleated and expand
- Friction from particles yield a terminal velocity ξ_w of the bubbles
- The bubble can run away when the friction is not enough to stop the bubble's acceleration



$$\nabla_\mu T_{\text{field}}^{\mu\nu} = \frac{\partial V}{\partial \phi} \partial^\nu \phi + \eta u^\mu \partial_\mu \phi \partial^\nu \phi,$$

$$\nabla_\mu T_{\text{fluid}}^{\mu\nu} = -\frac{\partial V}{\partial \phi} \partial^\nu \phi - \eta u^\mu \partial_\mu \phi \partial^\nu \phi,$$

Boltzmann equation

$$\left(p \cdot \partial + \frac{1}{2} \partial_\mu m_X^2 \partial_\mu \right) f_X(\vec{p}, x) \delta(p^2 - m^2) = -C[f_X]$$

$$\partial_\mu T_{\text{fluid}}^{\mu\nu} = -\partial^\nu \phi \frac{dm^2}{d\phi} \int \frac{d^3 p}{(2\pi)^3} \frac{1}{2E_{\vec{p}}} f(p, x) \Big|_{p^0=E_{\vec{p}}}$$

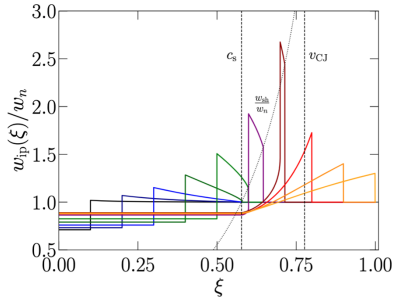
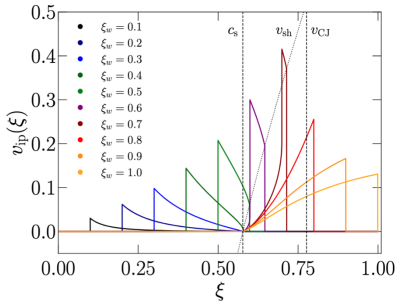


$$f_{\text{eq}} \longrightarrow \partial^\nu V_1(\phi, T)$$

$$\delta f \longrightarrow -\eta \delta^\nu \phi u^\mu \partial_\mu \phi$$

Hydrodynamics of first-order phase transitions⁷

- Depending on the wall velocity ξ_w and the strength of the phase transition $\alpha = \epsilon/\rho_{\text{rad}}$, we can have 3 types of solution: *subsonic deflagrations*, *supersonic deflagrations (hybrids)* or *detonations*.

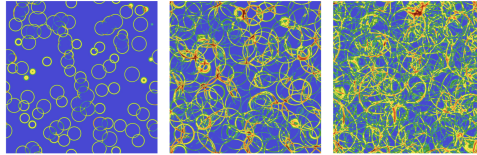


These calculations are available on CosmoGW, using
`pip install cosmoGW`

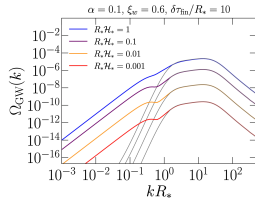
⁷ Espinosa, Konstandin, No, Servant, *JCAP* **06** (2010) 028.

GWs from sound waves⁸

- Numerical simulations of the scalar + fluid system performed by the Sussex/Helsinki group via an effective friction term indicate sound-wave regime to dominate for weak/intermediate phase transitions.



- Sound-shell model. Two scales are found that determine the GW spectrum: R_* and ΔR_* (sound-shell thickness).

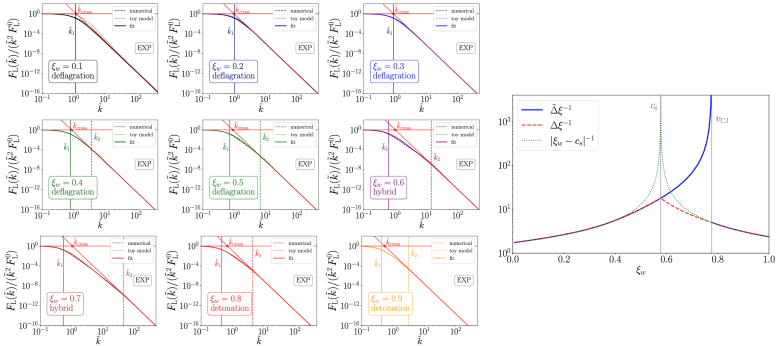


$$\Omega_{\text{GW}}(f) = 3 \tilde{\Omega}_{\text{GW}} K^2 \Upsilon(\tau_{\text{sw}})(H_* R_*) S(f R_*)$$

$$\Upsilon(\tau_{\text{sw}}) = \frac{H_* \tau_{\text{sw}}}{1 + H_* \tau_{\text{sw}}}$$

⁸Hindmarsh *et al.*, 2013, 2015, 2017, Cutting *et al.*, 2019, Correia *et al.*, 2025

GWs from sound waves⁹



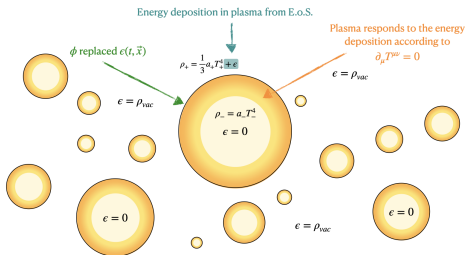
- Identified two peaks directly from the velocity spectrum: $\tilde{k}_1$ and $\tilde{k}_2$, determined by positions of discontinuities in the 1D profiles, and by nucleation history.

$$\frac{\tilde{k}_{1,2}^{\text{exp}}}{z_{1,2}} \simeq \{0.17, 0.21\}, \quad \frac{\tilde{k}_{1,2}^{\text{sim}}}{z_{1,2}} \simeq \{0.38, 0.25\}.$$

$$\xi_1 \equiv \frac{2\pi}{z_1} \simeq \begin{cases} \frac{4}{3}(\xi_f + \xi_w) & \text{for deflagrations,} \\ \frac{5}{3}(\xi_w + c_s) & \text{for detonations.} \end{cases} \quad \xi_2 \equiv \frac{2\pi}{z_2} \simeq \begin{cases} 2(\xi_f - \xi_w) & \text{for deflagrations,} \\ \frac{4}{3}(\xi_w - c_s) & \text{for detonations.} \end{cases}$$

GWs from sound waves: Higgsless simulations¹⁰

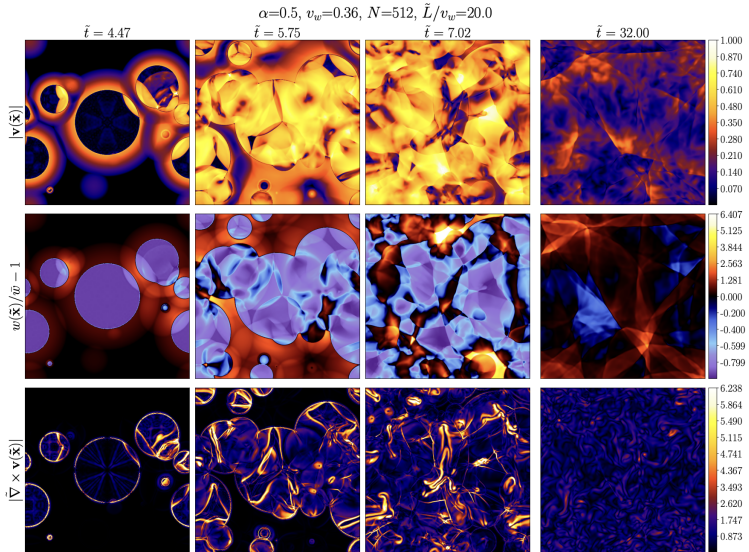
- Difficulty on simulations is due to the different scales of the scalar field ϕ and the fluid shell, so one can consider a nucleation history and set the pressure and energy density by knowing the value of ϵ and setting it during the simulation.
- Effect of bubble collisions on GWs is subdominant when sound waves are produced, so one can ignore the scalar field.
- Nucleation history is produced from an exponential probability distribution $P(t) \propto \exp[\beta(t - t_*)]$.



Credit: I. Stomberg

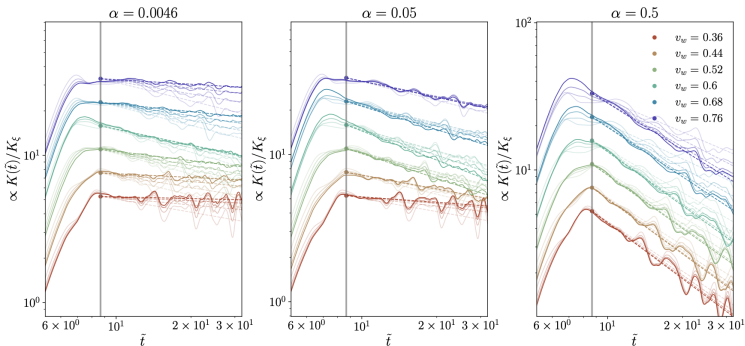
¹⁰ Jinno *et al.* JCAP 02 (2023) 011, 2209.04369,

Higgsless simulations of strong PTs¹¹



Higgsless simulations (results)¹²

- Kinetic energy decay is observed in the simulations.
- For weak and strong PTs, increasing discretization enhances the decay.
- Potential indication of the development of non-linearities (turbulence).



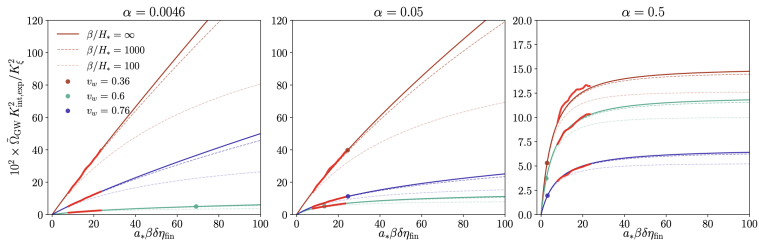
Higgsless simulations (results)¹³

- In the literature, the GW spectrum from sound waves is usually assumed to be

$$\Omega_{\text{GW}}(f) = 3 \tilde{\Omega}_{\text{GW}} K^2 \mathfrak{T}(\tau_{\text{sw}}) (H_* R_*) S(f R_*)$$

- The linear growth, which only appears when expansion is neglected, is modified when the decay of the source is significant (e.g., due to the development of non-linearities).
- Extended model to proposed locally stationary UETC

$$\Omega_{\text{GW}}(f) = 3 \tilde{\Omega}_{\text{GW}} \mathfrak{K}_{\text{int,exp}}^2 (H_* R_*) S(f R_*)$$



Higgsless simulations (results)¹⁰

- In the literature, the GW spectrum from sound waves is usually assumed to be

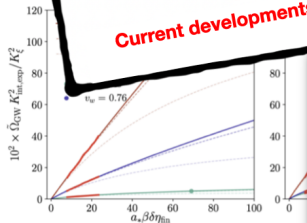
$$\Omega_{\text{GW}}(f) = 3 \tilde{\Omega}_{\text{GW}} K^2 \mathcal{T}(\tau_{\text{sw}}) (H_* R_*) S(\epsilon, \beta)$$

- The linear growth, which ends when the decay of the fluid motion is modified

- sources based on Higgsless simulations available using public code CosmoGW

Template for sound-shell model and template for non-linear
sources based on Higgsless simulations available using public
code CosmoGW
(<https://github.com/cosmoGW/cosmoGW>)
proceedings on arXiv with I. Stomberg (2508.04263) and tutorial
available on CosmoGW documentation

For use: pip install cosmoGW
Current developments undergoing to extend CosmoGW to
other sources



Gravitational wave spectra for cosmological phase transitions with non-linear decay of the fluid motion

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COSMOGW manual (coming soon)

Classical and Quantum Gravity

CONTRIBUTION TO “FOCUS ON THE LISA MISSION AT THE TIME OF ADOPTION”

COSMOGW: A public library to study cosmological backgrounds of gravitational waves. Part I — Phase transitions: Sound waves and MHD turbulence

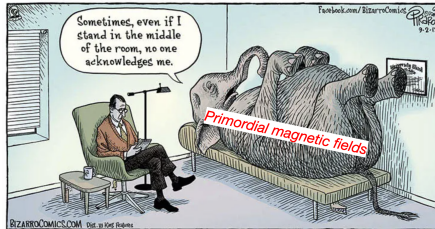
A. Roper Pol^{1,*}, M. Gurgenzidze^{2,3}, A. S. Midiri¹, M. Salomé^{1,4} and I. Stomberg⁵

Documentation available on Read the Docs, source code on GitHub.

- Routines to generate GW backgrounds from sound waves and phase transitions
- Reading routines from numerical simulations
- Cosmology calculations
- Interferometry, detector sensitivities

Primordial magnetic fields

- Magnetic fields can either be produced at or present during cosmological phase transitions.
- The magnetic fields are strongly coupled to the primordial plasma and effectively produce vortical motion, inevitably leading to the development of MHD turbulence.¹⁴
- Present magnetic fields can be amplified by primordial turbulence via dynamo.¹⁵



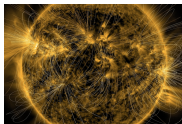
¹⁴ J. Ahonen and K. Enqvist, *Phys. Lett. B* **382**, 40 (1996).

¹⁵ A. Brandenburg *et al.* (incl. ARP), *Phys. Rev. Fluids* **4**, 024608 (2019).

Our magnetized Universe



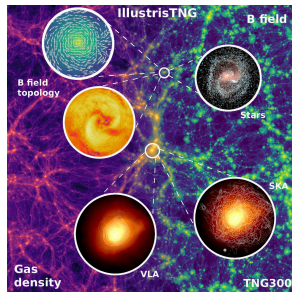
$\sim 1 \text{ G}$



(FIR 145 μm)



(Radio 6 cm Fletcher et. al)



Marinacci et al. (2017), Illustris TNG

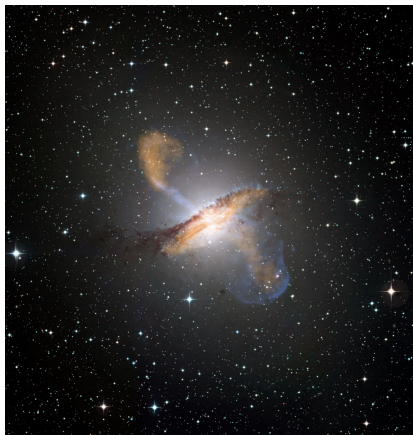
$\sim 10^{-5} \text{ G}$

Borlaff et al. (2021), SOFIA, M51

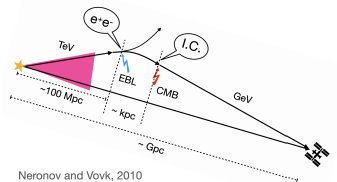
$\sim 10^{-9} - 10^{-6} \text{ G}$

- Dense regions in our Universe are strongly magnetized with $B \sim 10 \mu\text{G}$ in galaxies and clusters ($\sim 10 \text{ kpc}$). Radio and far-infrared (FIR) observations.
- Filaments also contain magnetic fields with $B \sim 10 \text{ nG}$ (LOFAR observations of Faraday rotation measurements).

Intergalactic magnetic fields in voids of the LSS



- γ -ray observations from distant blazars by Fermi/MAGIC collaboration show a removal of power at GeV, providing evidence for an intergalactic magnetic field.
- Lower bound of $B \sim 10^{-16}$ G at Mpc scales.



Neronov and Vovk, 2010

Relics from the early Universe?

- The observed intergalactic magnetic fields could have a **primordial** (from inflation or from a phase transition) or an **astrophysical** origin.
- Observations indicate a large volume filling factor in the voids and large correlation length scales (Mpc), favoring a primordial origin.
- Alternative possibilities to explain the blazar observations are:
 - *Beam-plasma instabilities* (suppressing the GeV emission) proposed¹⁶, but seems to not be significant at the intergalactic scales, *recently shown to be suppressed experimentally with the Super Proton Synchrotron at CERN*.¹⁷
 - *Magnetized galactic outflows*. Studied using cosmological simulations. However, it is difficult to reach large volume filling factors.¹⁸
- Primordial magnetic field present at recombination could alleviate the Hubble tension by reducing the sound horizon.¹⁹

¹⁶ Broderick et al. (2018).

¹⁷ Arrosmith et al. (2025).

¹⁸ Ni et al. (2024), Tjemsland et al. (2024).

¹⁹ Jedamzik & Pogosian (2020).

Generation of primordial magnetic fields during inflation

- Magnetic fields could be amplified from quantum fluctuations during inflation.
- For inflationary magnetogenesis to be viable, conformal invariance needs to be broken,²⁰ with $\mathcal{L} \sim f(\phi)F_{\mu\nu}F^{\mu\nu}$ or $g(\phi)F_{\mu\nu}\tilde{F}^{\mu\nu}$. Otherwise, magnetic field perturbations would decay with the Universe expansion.
- After reheating, magnetic fields are strongly coupled to the primordial plasma and effectively produce vortical motion, inevitably leading to the development of MHD turbulence.²¹
- Alternatively, small seed magnetic fields can be amplified by primordial turbulence induced by phase transitions, e.g., via dynamo.²²

²⁰Turner & Widrow (1988), Ratra (1992), Gasperini *et al.* (1995).

²¹J. Ahonen and K. Enqvist, *Phys. Lett. B* **382**, 40 (1996).

²²A. Brandenburg *et al.* (incl. ARP), *Phys. Rev. Fluids* **4**, 024608 (2019).

Generation of primordial magnetic fields during phase transitions

- Parity-violating processes during the electroweak phase transition ($T \sim 100$ GeV) are predicted by SM extensions that account for baryogenesis and can produce helical magnetic fields through sphaleron decay or B+L anomalies.²³

$$\mathbf{B} = \nabla \times \mathbf{A} - i \frac{2 \sin \theta_w}{g v^2} \nabla \Phi^\dagger \times \nabla \Phi$$

- Also after inflation, axion fields can amplify and produce magnetic fields.²⁴ For example, the QCD axion could oscillate and produce magnetic fields around the QCD scale ($T \sim 100$ MeV).²⁵

$$\mathcal{L} \supset \frac{\phi}{f} F_{\mu\nu} \tilde{F}^{\mu\nu}$$

²³T. Vachaspati, *Phys. Rev. B* **265**, 258 (1991), T. Vachaspati, *Phys. Rev. Lett.* **87**, 251302 (2001), J. M. Cornwall, *Phys. Rev. D* **56**, 6146 (1997).

²⁴M. M. Forbes and A. R. Zhitnitsky, *Phys. Rev. Lett.* **85**, 5268 (2000).

²⁵Miniati, Gregori, Reville & Sarkar (2018).

Generation of primordial magnetic fields from chiral anomalies

- Chiral magnetic effect in the early Universe at large temperatures $T > 80$ TeV can lead to the production of hypermagnetic fields.²⁶
- Proposed chiral magnetic effect relevant at smaller scales²⁷ leading to a chiral plasma instability and development of magnetic fields that undergo inverse cascading.²⁸
- Chiral magnetic effect also present for zero chiral chemical potential from local inhomogeneities.²⁹
- Chiral magnetic effect can be activated from an active source of chiral chemical potential (e.g. associated to baryogenesis at the electroweak scale)³⁰

²⁶ M. Joyce and M. E. Shaposhnikov, *PRL* **79**, 1193 (1997)

²⁷ A. Boyarsky *et al.* *PRL* **108**, 031301 (2012).

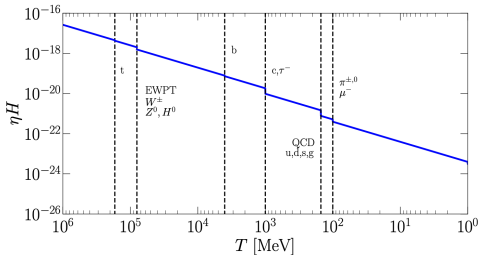
²⁸ I. Rogachevskii *et al.* *Astrophys. J. Lett* **845** (2017) L21, J. Schober *et al.* *Astrophys. J.* **858**, 124 (2018).

²⁹ J. Schober *et al.* *PRL* **132**, 6 (2024)

³⁰ M. Gurgunidze *et al.* (incl. ARP), arXiv:2512.09177 (2025).

GW sources in the early Universe

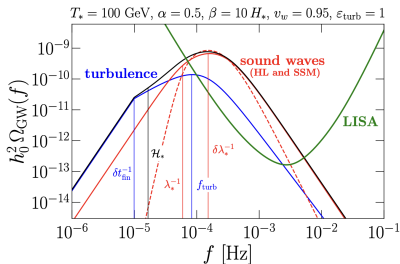
- Magnetohydrodynamic (MHD) sources of GWs:
 - Sound waves generated from first-order phase transitions.
 - (M)HD turbulence from first-order phase transitions.
 - Primordial magnetic fields.
- High-conductivity (low magnetic diffusivity $\eta = 1/\sigma$) of the early universe leads to a high-coupling between magnetic and velocity fields.³¹



GW sources in the early Universe

- Magnetohydrodynamic (MHD) sources of GWs:
 - Sound waves generated from first-order phase transitions.
 - (M)HD turbulence from first-order phase transitions.
 - Primordial magnetic fields.
- Other sources of GWs include
 - Bubble collisions.
 - Cosmic strings.
 - Primordial black holes.
 - Inflation.

ARP *et al.*, 2307.10744, 2308.12943



Primordial magnetic fields and GWs from MHD turbulence

- During the radiation-dominated era, after they are produced, magnetic fields will decay following MHD turbulence.
- Direct numerical simulations using the `PENCIL CODE`³² to solve:
 - ① Relativistic MHD equations adapted for radiation-dominated era (after electroweak symmetry is broken).
 - ② Gravitational waves equation.
- In general, large-scale simulations are necessary to solve the MHD nonlinearities (e.g., unequal-time correlators UETC and non-Gaussianities, which require simplifying assumptions in analytical studies).

³²Pencil Code Collaboration, JOSS **6**, 2807 (2020),
<https://github.com/pencil-code/>

MHD description

Right after the electroweak phase transition we can model the plasma using continuum MHD.

- Charge-neutral, electrically conducting fluid.
- Relativistic magnetohydrodynamic (MHD) equations.
- Radiation-dominated fluid

$$p = \rho c^2/3,$$

i.e., $c_s^2 = 1/3$ (ultrarelativistic EoS).

- Friedmann–Lemaître–Robertson–Walker metric

$$g_{\mu\nu} = \text{diag}\{-1, a^2, a^2, a^2\}$$

Conservation laws for MHD turbulence

$$T^{\mu\nu}_{;\nu} = 0, \quad F^{\mu\nu}_{;\nu} = -J^\mu, \quad \tilde{F}^{\mu\nu}_{;\nu} = 0$$

In the limit of subrelativistic bulk flow:

$$\gamma^2 \sim 1 + (v/c)^2 + \mathcal{O}(v/c)^4$$

Relativistic MHD equations are reduced to³³

$$\begin{aligned} \frac{\partial \ln \rho}{\partial t} &= -\frac{4}{3} (\nabla \cdot \mathbf{u} + \mathbf{u} \cdot \nabla \ln \rho) + \frac{1}{\rho} [\mathbf{u} \cdot (\mathbf{J} \times \mathbf{B}) + \eta J^2], \\ \frac{D\mathbf{u}}{Dt} &= \frac{\mathbf{u}}{3} (\nabla \cdot \mathbf{u} + \mathbf{u} \cdot \nabla \ln \rho) - \frac{\mathbf{u}}{\rho} [\mathbf{u} \cdot (\mathbf{J} \times \mathbf{B}) + \eta J^2] \\ &\quad - \frac{1}{4} \nabla \ln \rho + \frac{3}{4\rho} \mathbf{J} \times \mathbf{B} + \frac{2}{\rho} \nabla \cdot (\rho \nu \mathbf{S}), \\ \frac{\partial \mathbf{B}}{\partial t} &= \nabla \times (\mathbf{u} \times \mathbf{B} - \eta \mathbf{J}), \quad \mathbf{J} = \nabla \times \mathbf{B}, \end{aligned} \tag{1}$$

for a flat expanding universe with comoving and normalized

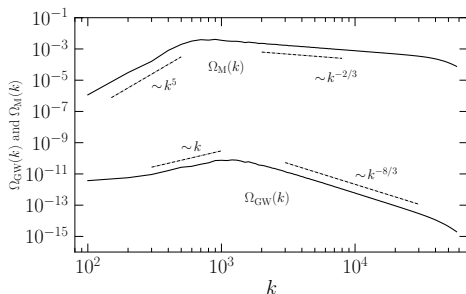
$\rho = a^4 \rho_{\text{phys}}$, $\rho = a^4 \rho_{\text{phys}}$, $B_i = a^2 B_{i,\text{phys}}$, u_i , and conformal time t ($dt = a dt_c$).

³³ A. Brandenburg, *et al.*, *Phys. Rev. D* **54**, 1291 (1996).

ARP, Midiri, *Relativistic Magnetohydrodynamics in the early Universe*, accepted in JCAP, arXiv:2501.05732 (2025).

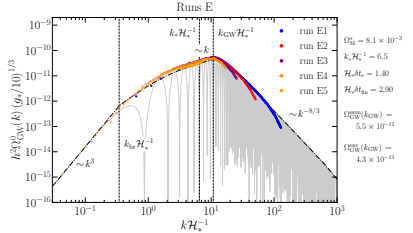
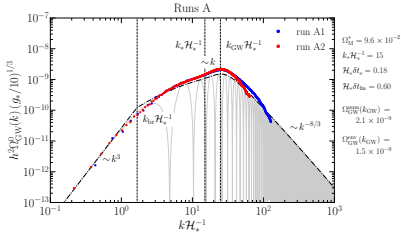
Numerical results for decaying MHD turbulence³⁴

$$1152^3, k_* = 2\pi \times 100, \Omega_M \sim 10^{-2}, \sigma_M = 1$$



- **Characteristic k scaling in the subinertial range for the GW spectrum.**
- k^2 expected at scales $k < k_*$ and k^3 at $k < H_*$ according to the “top-hat” model (Caprini *et al.*, 2020).

Numerical results for decaying MHD turbulence³⁵



run	Ω_{M}^*	$k_* \mathcal{H}_*^{-1}$	$\mathcal{H}_* \delta t_e$	$\mathcal{H}_* \delta t_{\text{fin}}$	$\Omega_{\text{GW}}^{\text{num}}(k_{\text{GW}})$	$[\Omega_{\text{GW}}^{\text{num}}/\Omega_{\text{GW}}^{\text{num}}](k_{\text{GW}})$	n	$\mathcal{H}_* L$	$\mathcal{H}_* t_{\text{end}}$	$\mathcal{H}_* \eta$
A1	9.6×10^{-2}	15	0.176	0.60	2.1×10^{-9}	1.357	768	6π	9	10^{-7}
A2	—	—	—	—	—	—	768	12π	9	10^{-6}
E1	8.1×10^{-3}	6.5	1.398	2.90	5.5×10^{-11}	1.184	512	4π	8	10^{-7}
E2	—	—	—	—	—	—	512	10π	18	10^{-7}
E3	—	—	—	—	—	—	512	20π	61	10^{-7}
E4	—	—	—	—	—	—	512	30π	114	10^{-7}
E5	—	—	—	—	—	—	512	60π	234	10^{-7}

Analytical model for GWs from decaying turbulence

- Assumption: magnetic or velocity field evolution $\delta t_e \sim 1/(u_* k_*)$ is slow compared to the GW dynamics ($\delta t_{\text{GW}} \sim 1/k$) at all $k \gtrsim u_* k_*$.
- We can derive an analytical expression for nonhelical fields of the envelope of the oscillations³⁶ of $\Omega_{\text{GW}}(k)$.

$$\Omega_{\text{GW}}(k, t_{\text{fin}}) \approx 3 \left(\frac{k}{k_*} \right)^3 \Omega_{\text{M}}^*{}^2 \frac{\mathcal{C}(\alpha)}{\mathcal{A}^2(\alpha)} p_{\Pi} \left(\frac{k}{k_*} \right) \\ \times \begin{cases} \ln^2[1 + \mathcal{H}_* \delta t_{\text{fin}}] & \text{if } k \delta t_{\text{fin}} < 1, \\ \ln^2[1 + (k/\mathcal{H}_*)^{-1}] & \text{if } k \delta t_{\text{fin}} \geq 1. \end{cases}$$

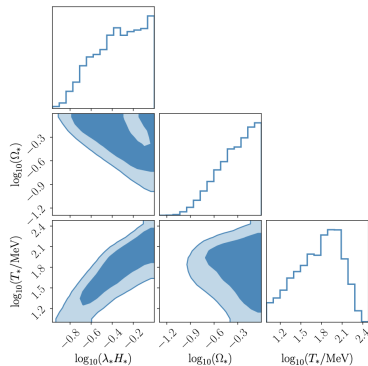
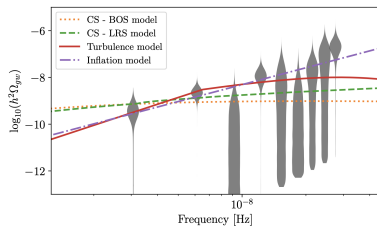
- p_{Π} is the anisotropic stress spectrum and depends on spectral shape, can be approximated for a von Kármán spectrum as³⁷

$$p_{\Pi}(k/k_*) \simeq \left[1 + \left(\frac{k}{2.2k_*} \right)^{2.15} \right]^{-11/(3 \times 2.15)}$$

³⁶ ARP et al., *Phys. Rev. D* **105**, 123502 (2022).

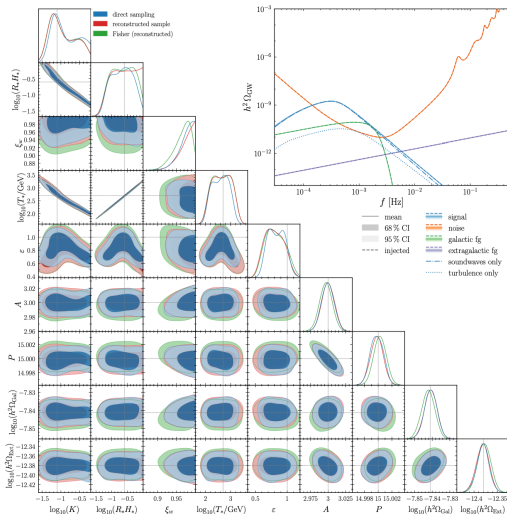
³⁷ ARP et al., arXiv:2307.10744 (2023).

Primordial magnetic fields constraints with PTA³⁸



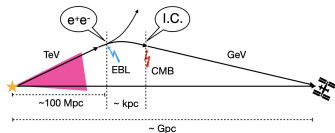
³⁸[EPTA collab.] (incl. ARP), arXiv:2306.16227 (2023).

Parameter reconstruction of GW signal for sound waves and turbulence from a PT with LISA³⁹

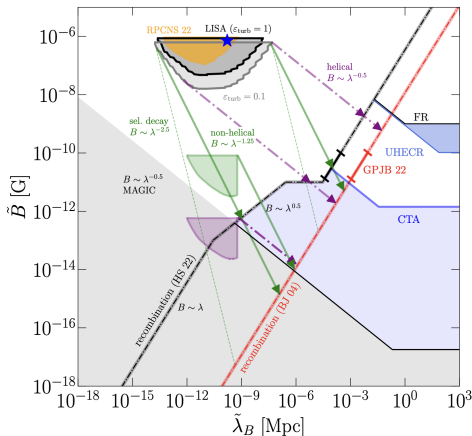


Multi-messenger studies of primordial magnetic fields³⁰

- Primordial magnetic fields would evolve through the history of the universe up to the present time and could explain the lower bounds in cosmic voids derived by the Fermi collaboration.³¹



- Maximum amplitude of primordial magnetic fields is constrained by the big bang nucleosynthesis.³²
- Additional constraints from CMB, Faraday Rotation, ultra-high energy cosmic rays (UHECR).



³⁰ ARP et al., arXiv:2307.10744 (2023).

³¹ A. Neronov and I. Vovk, *Science* **328**, 73 (2010).

³² V. F. Shvartsman, *Pisma Zh. Eksp. Teor. Fiz.* **9**, 315 (1969).

Conclusions

- Velocity and magnetic fields in the early universe can significantly contribute to the cosmological GW background (GWB) via sound waves and (M)HD turbulence.
- An observation of a GWB from the early Universe could probe BSM physics that are not accessible by other means or that are complementary to collider experiments.
- The GWB produced by non-linear fluid (and magnetic field) dynamics requires, in general, performing high-resolution numerical simulations, which can be done using open-source codes like `PENCIL CODE` or `COSMOLATTICE`.
- We expect the dynamics of first-order phase transitions to be affected by gauge fields, with its impact on the resulting GWB and on the fluid dynamics sourcing GWs being at the moment uncertain.
- Magnetic fields are ubiquitous in nature at all scales, from the smallest structures up to galaxies, cluster of galaxies, filaments, and potentially in cosmic voids of the LSS.
- Primordial magnetic fields produced in the early Universe can be probed by other cosmological (CMB) and astrophysical (γ -ray, radio observations at voids, filaments of the LSS), providing us a multi-messenger approach to study early Universe physics.
- To extract clean information from a GWB observation in LISA, PTA, or ET, we will need to have accurate modeling of the resulting GWB, since the reconstruction of the signal is very challenging (multiple cosmo sources, astrophysical background, detectors noise).



Thank You!

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github.com/AlbertoRoper/cosmoGW
albertoroper.github.io

