Gravitational waves from first-order phase transitions

Rencontres de Moriond Cosmology
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arXiv: 1903.08585, 2009.14174, 2201.05630, 2307.10744, 2308.12943, 2403.03723

`https://github.com/AlbertoRoper/cosmoGW` [CosmoGW]

Cosmological GW background

Cosmological GWs have the potential to provide us with *direct information on early universe physics* that is *not accessible via electromagnetic observations, possibly complementary to collider experiments*:

nature of first-order phase transitions
(baryogenesis, BSM physics, high-energy physics),
primordial origin of intergalactic magnetic fields.

Probing the early Universe with GWs

Cosmological (pre-recombination) GW background

- Why background? Individual sources are not resoluble, superposition of single events occurring in the whole Universe.

$$f_* \simeq 1.64 \times 10^{-3} \frac{100}{R_* \mathcal{H}_*} \frac{T_*}{100\,\mathrm{GeV}}\,\mathrm{Hz}$$

- Phase transitions
  - Ground-based detectors (LVK, ET, CE) frequencies are 10–1000 Hz
    Peccei-Quinn, B-L, left-right symmetries $\sim 10^7, 10^8$ GeV.
  - Space-based detectors **(LISA)** frequencies are 10^{-5}–10^{-2} Hz
    Electroweak phase transition $\sim$ 100 GeV
  - Pulsar Timing Array (PTA) frequencies are 10^{-9}–10^{-7} Hz
    Quark confinement (QCD) phase transition $\sim$ 100 MeV
- From inflation
  - B-modes of CMB anisotropies ($f_c \sim 10^{-18}$ Hz).
  - Can cover all f spectrum, depending on end-of-reheating T, and blue-tilted (beyond slow-roll inflation).

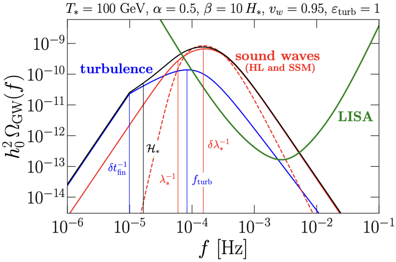
GW sources in the early universe

- Magnetohydrodynamic (MHD) sources of GWs:
  - Sound waves generated from first-order phase transitions.
  - Primordial magnetic fields.
  - (M)HD turbulence from first-order phase transitions.

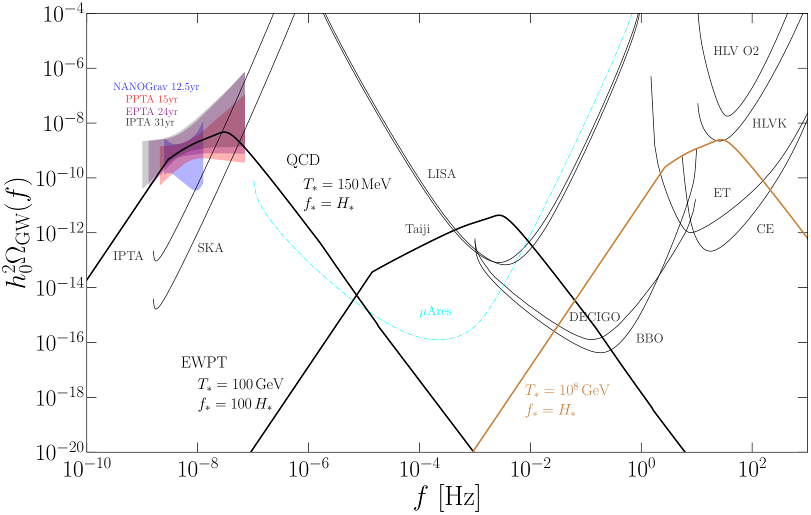
- High-conductivity of the early universe leads to a high-coupling between magnetic and velocity fields.

- Other sources of GWs include
  - Bubble collisions.
  - Cosmic strings.
  - Primordial black holes.
  - Inflation.

ARP *et al.*, 2307.10744, 2308.12943,
[LISA CosWG] (incl. ARP), arXiv:2403.03723



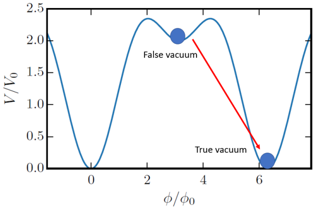
Gravitational spectrum (turbulence from PTs)[1]

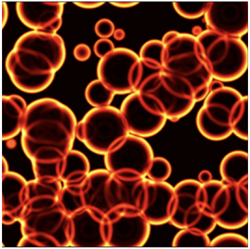


[1] ARP, C. Caprini, A. Neronov, D. Semikoz, *PRD* **105**, 123502 (2022)
A. Neronov, ARP, C. Caprini, D. Semikoz, *PRD* **103**, L041302 (2021)
ARP *et al.*, arXiv:2307.10744 (2023).

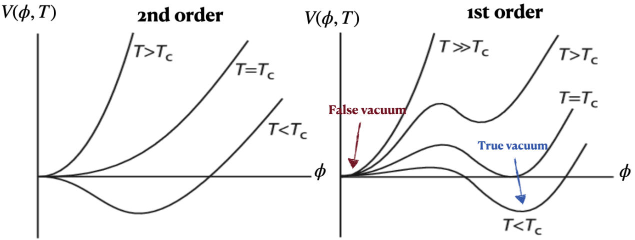
First-order phase transition

$$V(\phi, T) = \frac{1}{2}M^2(T)\phi^2 - \frac{1}{3}\delta(T)\phi^3 + \frac{1}{4}\lambda\phi^4$$



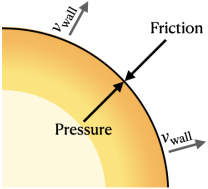


Credits: I. Stomberg



Hydrodynamics of first-order phase transitions[2]

- Broken-phase bubbles are nucleated and expand
- Friction from particles yield a terminal velocity ξ_w of the bubbles
- The bubble can run away when the friction is not enough to stop the bubble's acceleration

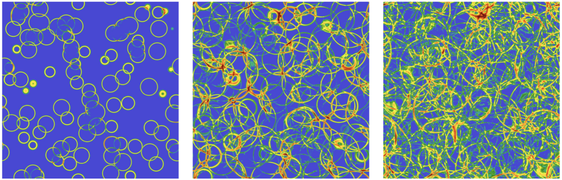


$$\nabla_\mu T^{\mu\nu}_{\text{field}} = \frac{\partial V}{\partial \phi}\partial^\nu \phi + \eta u^\mu \partial_\mu \phi \partial^\nu \phi\,,$$

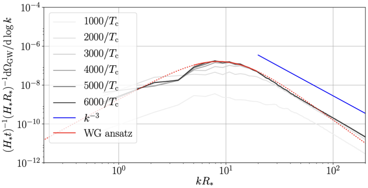
$$\nabla_\mu T^{\mu\nu}_{\text{fluid}} = -\frac{\partial V}{\partial \phi}\partial^\nu \phi - \eta u^\mu \partial_\mu \phi \partial^\nu \phi\,,$$

[2] Espinosa, Konstandin, No, Servant, *JCAP* **06** (2010) 028.

- Numerical simulations of the scalar + fluid system can be performed including an effective friction term



- Two scales are found that determine the GW spectrum: R_* and ΔR_* (sound-shell thickness).

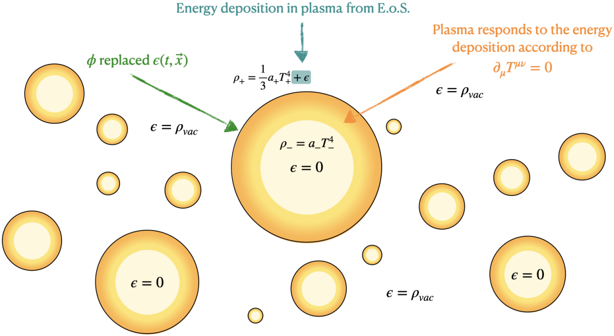


(b) Intermediate, $v_w = 0.92$

[3] Hindmarsh *et al.*, 2013, 2015, 2017.

GWs from sound waves: Higgsless simulations[4]

- Difficulty on simulations is due to the different scales of the scalar field ϕ and the fluid shell, so one can consider a nucleation history and set the pressure and energy density by knowing the value of ϵ and setting it during the simulation.
- Effect of bubble collisions on GWs is subdominant when sound waves are produced, so one can ignore the scalar field.



Credit: I. Stomberg

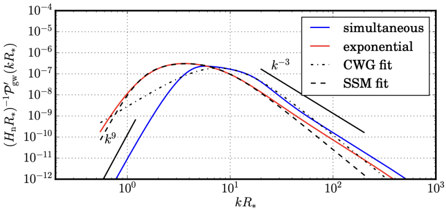
[4] Jinno *et al.*, 2020, 2022.

GWs from sound waves: Sound Shell Model[5]

- The sound shell model assumes linear superposition of velocity fields from each of the single bubbles and averages over nucleation locations and bubbles lifetimes (semi-analytical model), and the development of sound waves at the time of collisions.

$$\Omega_{\rm GW}(f) \propto S(f)\, K^2\, \tau_{\rm sw}\, R_*$$

- It predicts a steep k^9 spectrum and linear growth with time, according to HH19, and k^{-3} at large frequencies, with an intermediate k between $1/R_*$ and $1/\Delta R_*$.



(b) Intermediate, $v_{\rm w} = 0.92$

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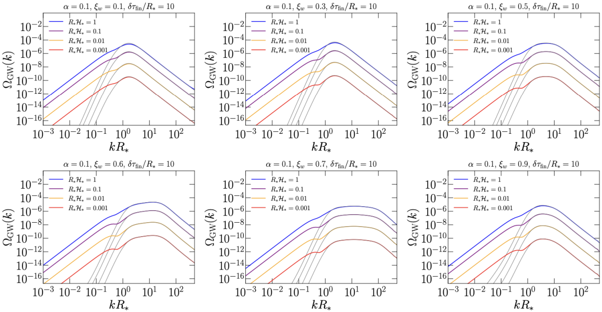
[5] Hindmarsh, 2016; Hindmarsh & Hijazi, 2019.

GWs from sound waves: Sound Shell Model revisited[6]

- Extended Sound Shell model to an expanding Universe and omitted assumptions that were not holding at small k.
- Recovered k^3 at small frequencies and found a $\ln^2(1 + \delta\tau_{\rm sw} H_*)$ time evolution of the causal branch and the "linear-in-time" evolution $\Upsilon = 1 - 1/(1 + \delta\tau_{\rm sw} H_*)$ around the peak, as well as a sharp bump.



6 ARP *et al.*, Phys. Rev. D, arXiv:2308.12943.

GWs from (M)HD turbulence

- Direct numerical simulations using the Pencil Code[7] to solve:
  1. Relativistic MHD equations adapted for radiation-dominated era (after electroweak symmetry is broken).
  2. Gravitational waves equation.

- In general, large-resolution simulations are necessary to solve the MHD nonlinearities (e.g., unequal-time correlators UETC and non-Gaussianities, which require simplifying assumptions in analytical studies).

[7] Pencil Code Collaboration, JOSS **6**, 2807 (2020), `https://github.com/pencil-code/`
ARP *et al., Geophys. Astrophys. Fluid Dyn.* **114**, 130 (2020).

Conservation laws for MHD turbulence

$$T^{\mu\nu}{}_{;\nu} = 0, \quad F^{\mu\nu}{}_{;\nu} = -J^{\mu}, \quad \tilde{F}^{\mu\nu}{}_{;\nu} = 0$$

In the limit of subrelativistic bulk flow:

$$\gamma^2 \sim 1 + (v/c)^2 + \mathcal{O}(v/c)^4$$

Relativistic MHD equations are reduced to[8]

$$\frac{\partial \ln \rho}{\partial t} = -\frac{4}{3}\left(\boldsymbol{\nabla} \cdot \boldsymbol{u} + \boldsymbol{u} \cdot \boldsymbol{\nabla} \ln \rho\right) + \frac{1}{\rho}\left[\boldsymbol{u} \cdot (\boldsymbol{J} \times \boldsymbol{B}) + \eta \boldsymbol{J}^2\right],$$

$$\frac{D\boldsymbol{u}}{Dt} = \frac{1}{3}\mathbf{u}\left(\boldsymbol{\nabla} \cdot \boldsymbol{u} + \boldsymbol{u} \cdot \boldsymbol{\nabla} \ln \rho\right) - \frac{\boldsymbol{u}}{\rho}\left[\boldsymbol{u} \cdot (\boldsymbol{J} \times \boldsymbol{B}) + \eta J^2\right]$$

$$-\frac{1}{4}\boldsymbol{\nabla} \ln \rho + \frac{3}{4\rho}\boldsymbol{J} \times \boldsymbol{B} + \frac{2}{\rho}\boldsymbol{\nabla} \cdot (\rho\nu \boldsymbol{S}),$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla} \times (\boldsymbol{u} \times \boldsymbol{B} - \eta \boldsymbol{J}), \quad \boldsymbol{J} = \boldsymbol{\nabla} \times \boldsymbol{B},$$

for a flat expanding universe with comoving and normalized $p = a^4 p_{\rm phys}, \rho = a^4 \rho_{\rm phys}, B_i = a^2 B_{i,{\rm phys}}, u_i$, and conformal time t $(\mathrm{d}t = a\mathrm{d}t_c)$.

[8] A. Brandenburg, *et al.*, *Phys. Rev. D* **54**, 1291 (1996).

Numerical results for decaying MHD turbulence[9]

Initial conditions

- Initial stochastic magnetic or (purely vortical) velocity field.

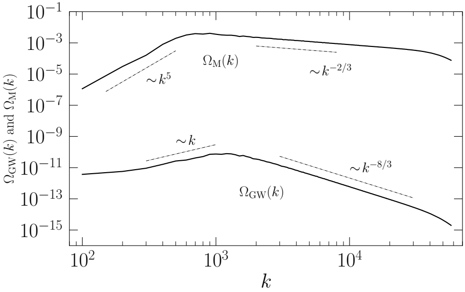
$$kB_i(\boldsymbol{k}) = \left(\delta_{ij} - \hat{k}_i \hat{k}_j\right) g_j \sqrt{2\Omega_{\rm M}(k)/k}$$

- Batchelor spectrum for magnetic (or vortical velocity) fields, i.e., $\Omega_{\rm M}(k) \equiv {\rm d}\rho_{\rm M}/{\rm d}\ln k \propto k^5$ for small $k < k_* \sim \mathcal{O}(\xi_{\rm M}^{-1})$.
- Kolmogorov spectrum in the inertial range, i.e., $\Omega_{\rm M} \propto k^{-2/3}$.

[9] A. Brandenburg *et al.* (incl. ARP), *Phys. Rev. D* **96**, 123528 (2017).
ARP *et al.*, *Phys. Rev. D* **102**, 083512 (2020).
ARP *et al.*, *JCAP* **04** (2022), 019.
ARP *et al.*, *Phys. Rev. D* **105**, 123502 (2022).

Numerical results for decaying MHD turbulence[10]

$1152^3, k_* = 2\pi \times 100, \Omega_{\rm M} \sim 10^{-2}, \sigma_{\rm M} = 1$



- **Characteristic k scaling in the subinertial range for the GW spectrum.**
- k^2 expected at scales $k < k_*$ and k^3 at $k < H_*$ according to the "top-hat" model (Caprini *et al.*, 2020).

[10] ARP *et al.*, *Phys. Rev. D* **102**, 083512 (2020).

Analytical model for GWs from decaying turbulence

- Assumption: magnetic or velocity field evolution $\delta t_{\rm e} \sim 1/(u_* k_*)$ is slow compared to the GW dynamics ($\delta t_{\rm GW} \sim 1/k$) at all $k \gtrsim u_* k_*$.
- We can derive an analytical expression for nonhelical fields of the envelope of the oscillations[11] of $\Omega_{\rm GW}(k)$.

$$\Omega_{\rm GW}(k, t_{\rm fin}) \approx 3 \left(\frac{k}{k_*}\right)^3 {\Omega_{\rm M}^*}^2 \frac{\mathcal{C}(\alpha)}{\mathcal{A}^2(\alpha)} \, p_\Pi\left(\frac{k}{k_*}\right) \times \begin{cases} \ln^2[1 + \mathcal{H}_* \delta t_{\rm fin}] & \text{if } k\, \delta t_{\rm fin} < 1, \\ \ln^2[1 + (k/\mathcal{H}_*)^{-1}] & \text{if } k\, \delta t_{\rm fin} \geq 1. \end{cases}$$

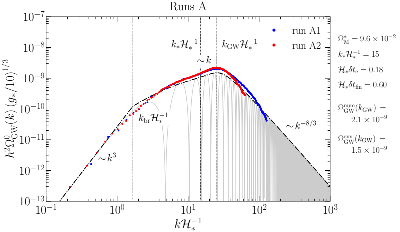
- p_Π is the anisotropic stress spectrum and depends on spectral shape, can be approximated for a von Kárman spectrum as[12]

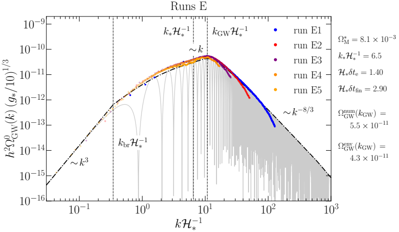
$$p_\Pi(k/k_*) \simeq \left[1 + \left(\frac{k}{2.2 k_*}\right)^{2.15}\right]^{-11/(3\times 2.15)}$$

[11] ARP *et al.*, *Phys. Rev. D* **105**, 123502 (2022).

[12] ARP *et al.*, arXiv:2307.10744 (2023).

Numerical results for nonhelical decaying MHD turbulence[13]

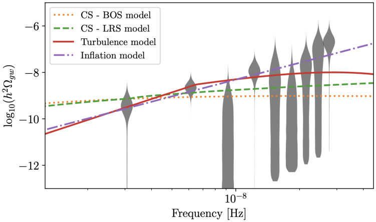


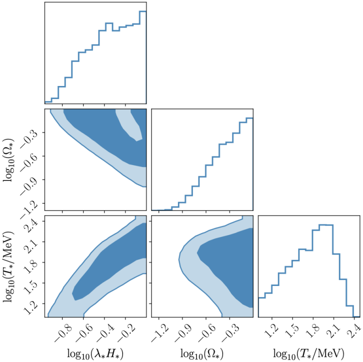


run	$\Omega_{\rm M}^*$	$k_*\mathcal{H}_*^{-1}$	$\mathcal{H}_*\delta t_e$	$\mathcal{H}_*\delta t_{\rm fin}$	$\Omega_{\rm GW}^{\rm num}(k_{\rm GW})$	$[\Omega_{\rm GW}^{\rm env}/\Omega_{\rm GW}^{\rm num}](k_{\rm GW})$	n	$\mathcal{H}_*L$	$\mathcal{H}_*t_{\rm end}$	$\mathcal{H}_*\eta$
A1	9.6×10^{-2}	15	0.176	0.60	2.1×10^{-9}	1.357	768	6π	9	10^{-7}
A2	–	–	–	–	–	–	768	12π	9	10^{-6}
E1	8.1×10^{-3}	6.5	1.398	2.90	5.5×10^{-11}	1.184	512	4π	8	10^{-7}
E2	–	–	–	–	–	–	512	10π	18	10^{-7}
E3	–	–	–	–	–	–	512	20π	61	10^{-7}
E4	–	–	–	–	–	–	512	30π	114	10^{-7}
E5	–	–	–	–	–	–	512	60π	234	10^{-7}

[13] ARP *et al.*, *Phys. Rev. D* **105**, 123502 (2022).

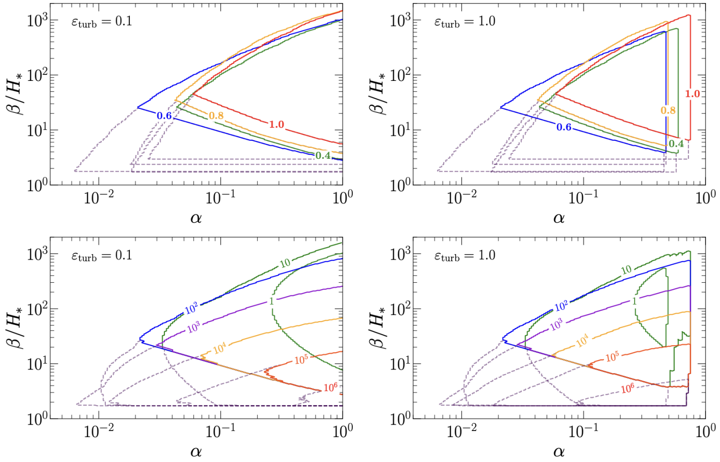
Primordial turbulence constraints with EPTA DR 2[14]





[14] [EPTA+InPTA] (incl. ARP), arXiv:2306.16227 (2023).

First-order phase transition parameters with LISA for sound waves + turbulence[15]



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15 ARP *et al.*, arXiv:2307.10744 (2023).

Conclusions

- Velocity and magnetic fields in the early universe can significantly contribute to the stochastic GW background (SGWB) via sound waves and (M)HD turbulence.
- MHD requires, in general, performing high-resolution numerical simulations, which can be done using the Pencil Code.
- Since the SGWB is a superposition of different sources, it is extremely important to characterize the different sources, to be able to extract clean information from the early universe physics.
- The interplay between sound waves and the development of turbulence is not well understood. It plays an important role on the relative amplitude of both sources of GWs.
- LISA, PTA, and next-generation ground-based detectors can potentially be used to probe the origin of magnetic fields in the largest scales of our Universe, which is still an open question in cosmology.
- Bubble nuccleation, sound wave production, and magnetogenesis physics can be coupled to our equations for more realistic production analysis (future work).

Thank You!

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github.com/AlbertoRoper/cosmoGW
cosmology.unige.ch/users/alberto-roper-pol