Gravitational waves from (non-linear compressional motion in) strong phase transitions

12th LISA CosWG workshop (July 22, 2026)
Estonian Academy of Sciences, Tallinn

Alberto Roper Pol
University of Geneva

SNSF Ambizione grant (2023–2027): "*Exploring the early universe with gravitational waves and primordial magnetic fields.*"

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Caprini, Jinno, Konstandin, ARP*, Rubira, Stomberg*, 2409.03651

ARP, Procacci, Caprini, 2308.12943

CosmoGW: `https://github.com/CosmoGW/CosmoGW`

Probing the early Universe with GWs

Cosmological (pre-recombination) GW background

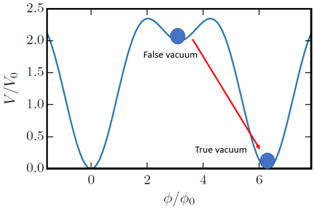
- Why background? Individual sources are not resoluble, superposition of single events occurring in the whole Universe.

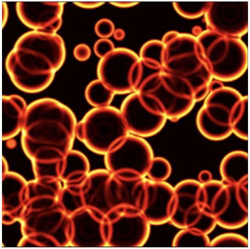
$$f_* \simeq 1.64 \times 10^{-3} \frac{100}{R_* \mathcal{H}_*} \frac{T_*}{100\,\mathrm{GeV}}\,\mathrm{Hz}$$

- Phase transitions
  - Ground-based detectors (LVK, ET, CE) frequencies are 10–1000 Hz
    Peccei-Quinn, B-L, left-right symmetries $\sim 10^7, 10^8$ GeV.
  - Space-based detectors **(LISA)** frequencies are 10^{-5}–10^{-2} Hz
    Electroweak phase transition $\sim$ 100 GeV
  - Pulsar Timing Array (PTA) frequencies are 10^{-9}–10^{-7} Hz
    Quark confinement (QCD) phase transition $\sim$ 100 MeV

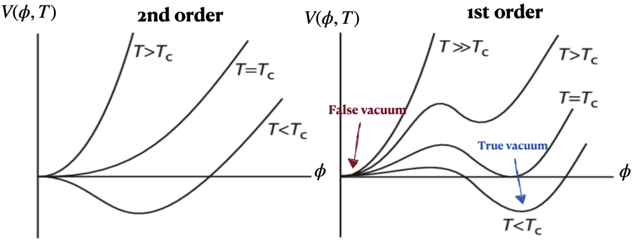
First–order phase transition

$$V(\phi, T) = \frac{1}{2}M^2(T)\phi^2 - \frac{1}{3}\delta(T)\phi^3 + \frac{1}{4}\lambda\phi^4$$



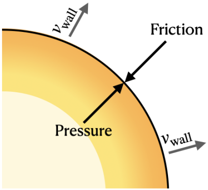


Credits: I. Stomberg



Hydrodynamics of first-order phase transitions[1]

- Broken-phase bubbles are nucleated and expand
- Friction from particles yield a terminal velocity ξ_w of the bubbles
- The bubble can run away when the friction is not enough to stop the bubble's acceleration



$$\nabla_\mu T^{\mu\nu}_{\rm field} = \frac{\partial V}{\partial \phi}\partial^\nu \phi + \eta u^\mu \partial_\mu \phi \partial^\nu \phi \,,$$

$$\nabla_\mu T^{\mu\nu}_{\rm fluid} = -\frac{\partial V}{\partial \phi}\partial^\nu \phi - \eta u^\mu \partial_\mu \phi \partial^\nu \phi \,,$$

[1] Espinosa, Konstandin, No, Servant, *JCAP* **06** (2010) 028.

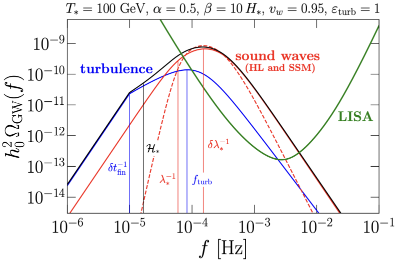
GW sources in the early Universe

- Magnetohydrodynamic (MHD) sources of GWs:
  - **Sound waves (compressional motion)**.
  - (M)HD turbulence (vortical motion).
  - Primordial magnetic fields.

- High-conductivity of the early universe leads to a high-coupling between magnetic and velocity fields.

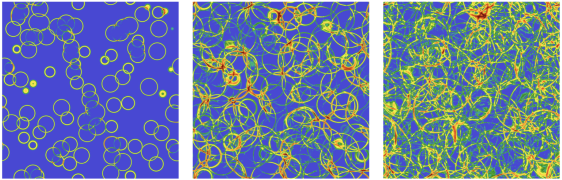
- Other sources of GWs include
  - Bubble collisions.
  - Cosmic strings.
  - Primordial black holes.

ARP *et al.*, 2307.10744, 2308.12943

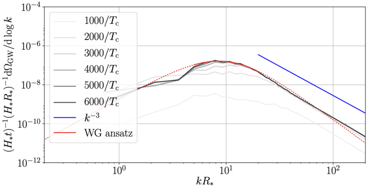


GWs from sound waves

- Numerical simulations of the scalar + fluid system performed by the Sussex/Helsinki group via an effective friction term.[2]



- Two scales are found that determine the GW spectrum: R_* and ΔR_* (sound-shell thickness).



(b) Intermediate, $v_w = 0.92$

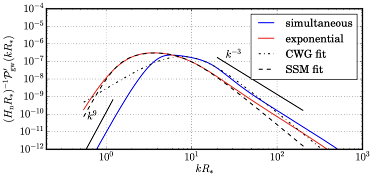
[2] Hindmarsh *et al.*, 2013, 2015, 2017, Cutting *et al.*, 2019, Correia *et al.*, 2025.

GWs from sound waves: Sound Shell Model[4]

- The sound shell model assumes linear superposition of velocity fields from each of the single bubbles and averages over nucleation locations and bubble lifetimes (semi-analytical model), and the development of sound waves at the time of collisions. It assumes stationary UETC $P_\Pi = P_\Pi(k, t_2 - t_1)$.

$$\Omega_{\rm GW}(f) = 3\,\tilde\Omega_{\rm GW}\,K^2\,(H_*\tau_{\rm sw})\,(H_* R_*)\,S(f\,R_*)$$

- It predicted a steep k^9 spectrum and linear growth with time, according to HH19, and k^{-3} at large frequencies, with an intermediate k between $1/R_*$ and $1/\Delta R_*$.
- GW predictions usually assume $\tau_{\rm sw} = \min(\tau_{\rm sh}, H_*^{-1})$, with $\tau_{\rm sh} \sim R_*/\sqrt{K}$ being the expected time to develop non-linearities (should be a conformal time interval $\tau_{\rm sw} = \tau_{\rm fin} - \tau_*$ due to the conformal invariance of the fluid equations![3]).



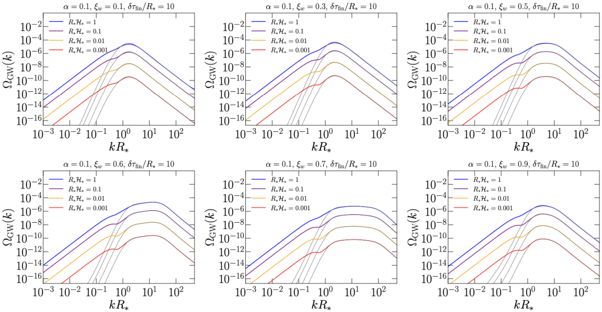
(b) Intermediate, $v_{\rm w} = 0.92$

[3] ARP & Midiri, 2025.

[4] Hindmarsh, 2016; Hindmarsh & Hijazi, 2019.

GWs from sound waves: Sound Shell Model revisited[5]

- Extended Sound Shell model to an expanding Universe and omitted assumptions that were not holding at small k. Furthermore, an additional contribution to the GW spectrum is identified, omitted in previous studies.
- Recovered k^3 at small frequencies and found a $\ln^2(1 + \tau_{\rm sw} H_*)$ time evolution of the causal branch and the "linear-in-time" evolution $\Upsilon = \tau_{\rm sw} H_* / (1 + \tau_{\rm sw} H_*)$ around the peak, as well as a sharp bump.

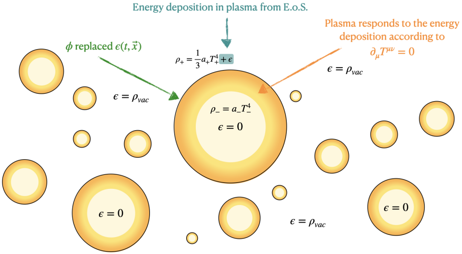


[5] ARP, Procacci, Caprini, Phys. Rev. D, arXiv:2308.12943

Sharma *et al.*, JCAP 12 (2023) 042.

GWs from sound waves: Higgsless simulations[6]

- Difficulty on simulations is due to the different scales of the scalar field ϕ and the fluid shell, so one can consider a nucleation history and set the pressure and energy density by knowing the value of ϵ and setting it during the simulation.
- Effect of bubble collisions on GWs is subdominant when sound waves are produced, so one can ignore the scalar field.
- Nucleation history is produced from an exponential probability distribution $P(t) \propto \exp\left[\beta(t - t_*)\right]$.



Credit: I. Stomberg

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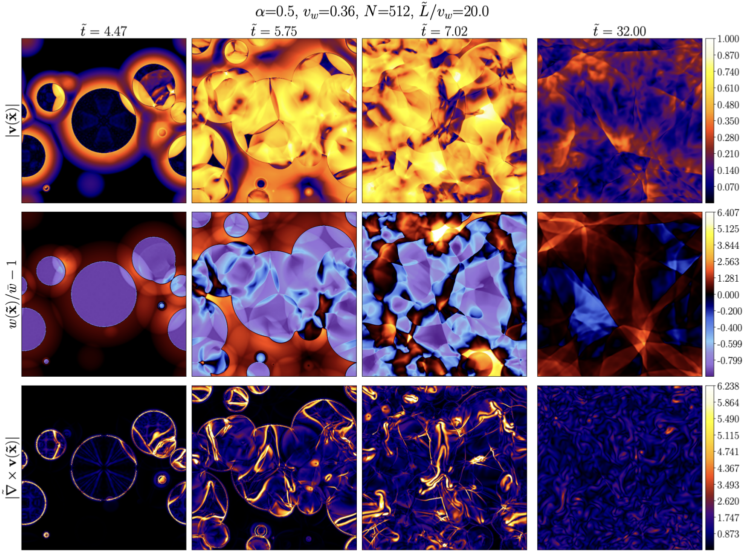
[6] Jinno *et al.* JCAP 02 (2023) 011, 2209.04369,

Caprini, Jinno, Konstandin, ARP, Rubira, Stomberg, arXiv:2409.03651

Higgsless simulations of strong PTs[7]



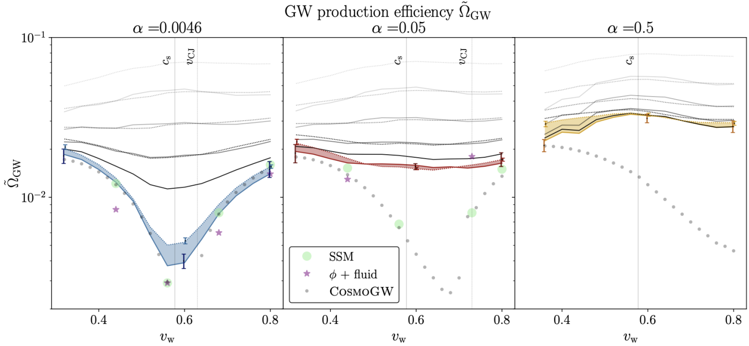
[7] Caprini, Jinno, Konstandin, ARP, Rubira, Stomberg, arXiv:2409.03651

Higgsless simulations (results)[8]

- In the literature, the GW spectrum from sound waves is usually assumed to be

$$\Omega_{\rm GW}(f) = 3\,\tilde{\Omega}_{\rm GW}\,K^2\,(H_*\tau_{\rm sw})\,(H_* R_*)\,S(f\,R_*)$$

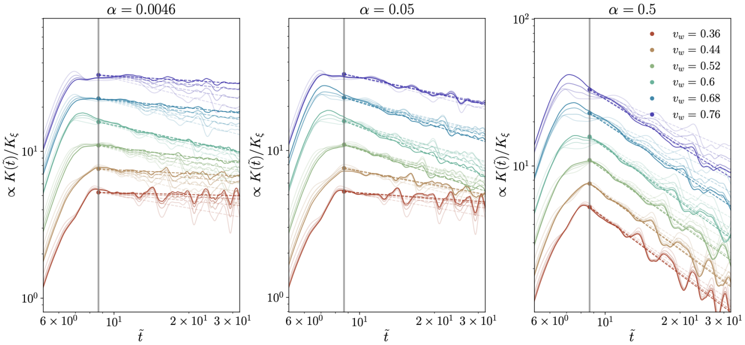
- $\tilde{\Omega}_{\rm GW}$ is the efficiency factor



[8] Caprini, Jinno, Konstandin, ARP, Rubira, Stomberg, arXiv:2409.03651

Higgsless simulations (results)[9]

- Kinetic energy decay is observed in the simulations.
- For weak and strong PTs, increasing discretization enhances the decay.
- Potential indication of the development of non-linearities (turbulence).



[9] Caprini, Jinno, Konstandin, ARP, Rubira, Stomberg, arXiv:2409.03651

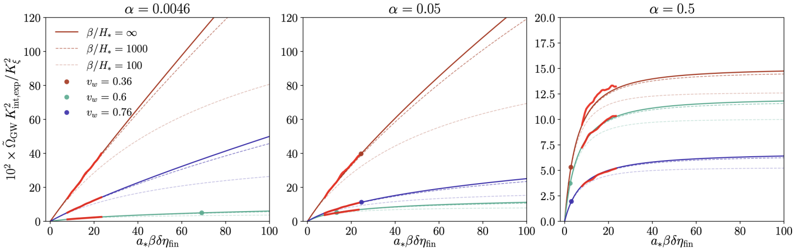
Higgsless simulations (results)[10]

- In the literature, the GW spectrum from sound waves is usually assumed to be

$$\Omega_{\rm GW}(f) = 3\,\tilde{\Omega}_{\rm GW}\,K^2\,(\mathbf{H}_*\tau_{\rm sw})\,(H_* R_*)\,S(f\,R_*)$$

- The linear growth, which only appears when expansion is neglected, is modified when the decay of the source is significant (e.g., due to the development of non-linearities).
- Extended model to proposed locally stationary UETC

$$\Omega_{\rm GW}(f) = 3\,\tilde{\Omega}_{\rm GW}\,\mathbf{K}^2_{\rm int}\,(H_* R_*)\,S(f\,R_*)$$



[10] Caprini, Jinno, Konstandin, ARP, Rubira, Stomberg, arXiv:2409.03651

CosmoGW code: a public tool for cosmological GW backgrounds

Routines

The main routines of cosmoGW are stored under src/cosmoGW:

- cosmoGW.py: functions relevant for cosmological stochastic gravitational wave backgrounds (SGWB).
- cosmology.py: functions relevant for cosmological calculations, including a Friedmann equations solver (see tutorial on Friedmann equations in cosmology.ipnyb) that can generate the solution files being read in some Pencil Code simulations (see tutorial cosmology_PC.ipnyb).
- cosmoMF.py: functions relevant for cosmological magnetic fields like bounds from different experiments, observations or projected sensitivities, and expectations from theory, among others. Coming soon!
- GW_analytical.py: contains analytical calculations and useful mathematical functions
- GW_models.py: models to describe the GW background produced from magnetic and velocity field perturbations in the primordial plasma, e.g., induced by a first-order phase transition. It includes models to describe the GW background from MHD turbulence and from compressional motion.
- GW_templates.py: contains templates to describb the GW background from different sources
- hydro_bubbles.py: functions to compute fluid perturbations induced by the expansion of bubbles in first-order phase transitions
- interferometry.py: functions to compute the response and sensitivity functions of interferometer space-based GW detectors (e.g., LISA and Taiji) to the detection of SGWBs (see tutorial on LISA interferometry in interferometry.ipynb) energy density and polarization, including the space-based network LISA-Taiji to detect polarization.
- modified_grav.py: functions relevant for GW production in the context of general theories of modified gravity. Coming soon!
- pta.py: functions used in the analysis of observations by pulsar timing array (PTA) collaborations: NANOGrav, PPTA, EPTA, and IPTA.
- reading.py: functions to read the output files of a specific set of runs (project) of the Pencil Code.
- spectra.py: contains description for specific spectral templates, postprocessing routines for numerical spectra, and other mathematical routines.

CosmoGW code: a public tool for cosmological GW backgrounds

- First stable version (v1.0) will be released by the end of the month, together with a manual, including:
  - Sound-shell model calculations for GW backgrounds.
  - Extension for non-linear compressional motion, following the findings of the Higgsless simulations.
  - Semi-analytical models for MHD turbulence, validated by Pencil Code simulations.
- Installable via pip: `pip install cosmoGW`
- Available at `https://pypi.org/project/cosmoGW/`

Conclusions

- Fluid perturbations induced by first-order phase transitions in the early universe can significantly contribute to the stochastic GW background (SGWB) via compressional (e.g. sound waves) and vortical (M)HD turbulence.
- Since the SGWB is a superposition of different sources, it is extremely important to characterize the different sources, to be able to extract clean information from the early universe physics.
- Numerical simulations are crucial to provide insights on the theoretical understanding and on the development of an analytical framework to provide useful and accurate templates for LISA.
- The interplay between sound waves (acoustic motion) and the development of turbulence is not yet well understood. It plays an important role on the relative amplitude of both sources of GWs. On-going and future simulations are required to understand it.

Thank You!

alberto.roperpol@unige.ch

github.com/CosmoGW/CosmoGW
cosmology.unige.ch/users/alberto-roper-pol

1st Pencil Code school on early Universe physics and gravitational waves (Oct 20-24)

The Pencil Code school on early Universe physics and gravitational waves will take place on October 20-24 as part of a two-week CERN TH institute.

The school targets early-career and senior researchers that are interested in learning and developing numerical skills applied to early Universe physics using Pencil Code.

The lectures will cover numerical aspects:

- *Introduction to Pencil Code*
- *Finite-difference schemes for partial differential equations*
- *Post-processing of data with IDL and Python*
- *GPU acceleration of Pencil Code*

as well as applications to particular physics cases with hands-on exercises on:

- *Magnetohydrodynamics of the early Universe*
- *Generation and evolution of primordial magnetic fields*
- *Chiral magnetohydrodynamics*
- *First-order phase transitions*
- *Gravitational wave production*
- *Axion inflation*

Registration is open and will close on ***July 31st***. The school is limited to a maximum of 30 participants.

Participants of the school are encouraged to also participate in the user meeting (Oct 27-31) and need to register separately.

CERN
Oct 20-24 (school) and Oct 27-31 (meeting)

https://indico.cern.ch/e/PCUM2025

Lecturers:

- Axel Brandenburg (Nordita)
- Philippe Bourdin (University of Graz)
- Simon Candelaresi (University of Augsburg)
- Deepen Garg (University of Bonn)
- Frederik Gent (Aalto University & Nordita)
- Matthias Rheinhardt (Aalto University)
- Alberto Roper Pol (University of Geneva)
- Isak Stomberg (IFIC, Valencia)

21st Pencil Code user meeting PCUM2025 (Oct 27-31)

The Pencil Code user meeting will take place on October 27-31 as part of a two-week CERN TH institute.

Registration for the Pencil Code user meeting is open and will close on ***September 28th***.