Gravitational waves from phase transitions in the early Universe: sound waves and MHD turbulence

Instituto de Física Teórica (May 4, 2026)

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Cosmological Gravitational Waves

- Gravity is the *weakest fundamental force*. Hence, GWs are difficult to detect but they propagate freely carrying *clean information of the source.*

- Cosmological GWs from the early Universe have the potential to provide us with *direct information on early universe physics* that is *not accessible via electromagnetic observations, complementary to collider experiments*.

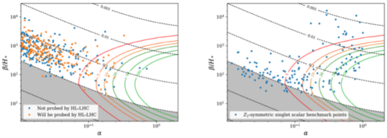
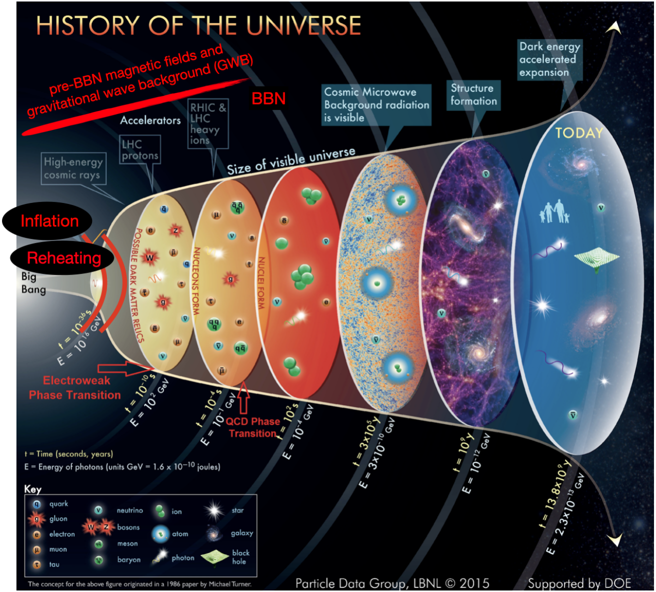


Figure 4: LEFT: Predicted values of α and β/H for $m_2 = 170$ GeV and 240 GeV (combined in one plot) in the general singlet model obtained by linearly varying the free parameters of the model and imposing requirements as described in the text. The mixing angles considered were $\sin\theta = 0.01$ (blue points) and 0.1 (orange points). The models in blue (orange) are unlikely (likely) to be probed by the high-luminosity LHC. The expected LISA sensitivities correspond to $T_* = 50$ GeV. RIGHT: Sensitivity curve for the Z_2 symmetric singlet extension. In both the left and right panels we have taken $v_w = 1$.

From [LISA CosWG], Caprini et al., *Detecting gravitational waves from cosmological phase transitions with LISA: an update* (2019)



HISTORY OF THE UNIVERSE
pre-BBN magnetic fields and gravitational wave background (GWB)
BBN
Accelerators
RHIC & LHC heavy ions
LHC protons
High-energy cosmic rays
Cosmic Microwave Background radiation is visible
Structure formation
Dark energy accelerated expansion
TODAY
Size of visible universe
Inflation
Reheating
Big Bang
POSSIBLE DARK MATTER RELICS
NUCLEONS FORM
NUCLEI FORM
Electroweak Phase Transition
QCD Phase Transition
t = 10⁻³⁸ s
E = 10¹⁶ GeV
t = 10⁻¹⁰ s
E = 10² GeV
t = 10⁻⁴ s
E = 10⁻¹ GeV
t = 10² s
E = 10⁻⁴ GeV
t = 3x10⁵ y
E = 3x10⁻¹⁰ GeV
t = 10⁹ y
E = 10⁻¹² GeV
t = 13.8x10⁹ y
E = 2.3x10⁻¹³ GeV
t = Time (seconds, years)
E = Energy of photons (units GeV = 1.6 x 10⁻¹⁰ joules)
Key
quark
gluon
electron
muon
tau
neutrino
bosons
meson
baryon
ion
atom
photon
star
galaxy
black hole
The concept for the above figure originated in a 1986 paper by Michael Turner.
Particle Data Group, LBNL © 2015
Supported by DOE

Probing the early Universe with GWs

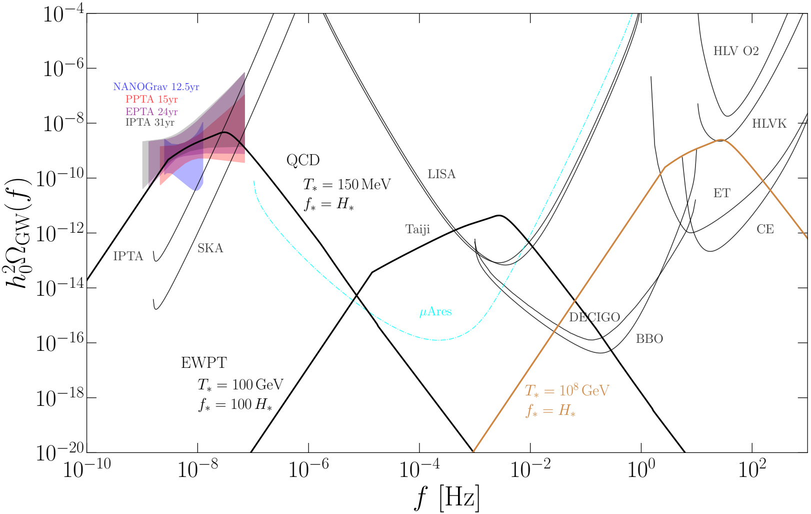
Cosmological (pre-recombination) GW background

- Why background? Individual sources are not resoluble, superposition of single events occurring in the whole Universe.

$$f_* \simeq 1.64 \times 10^{-3} \frac{0.01}{R_* \mathcal{H}_*} \frac{T_*}{100\,\mathrm{GeV}}\,\mathrm{Hz}$$

- Phase transitions
  - Ground-based detectors (LVK, ET, CE) frequencies are 10–1000 Hz
    Peccei-Quinn, B-L, left-right symmetries $\sim 10^7, 10^8$ GeV.
  - Space-based detectors **(LISA)** frequencies are 10^{-5}–10^{-2} Hz
    Electroweak phase transition $\sim$ 100 GeV
  - Pulsar Timing Array **(PTA)** frequencies are 10^{-9}–10^{-7} Hz
    Quark confinement (QCD) phase transition $\sim$ 100 MeV

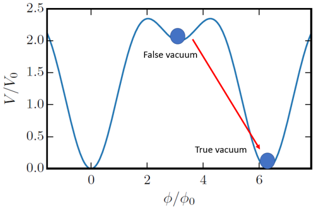
Gravitational spectrum from MHD turbulence and detectors[1]

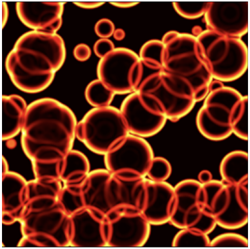


[1] ARP, C. Caprini, A. Neronov, D. Semikoz, *PRD* **105**, 123502 (2022)
A. Neronov, ARP, C. Caprini, D. Semikoz, *PRD* **103**, L041302 (2021)
ARP *et al.*, arXiv:2307.10744 (2023).

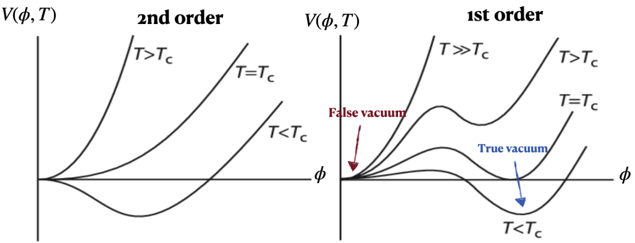
First–order phase transition

$$V(\phi, T) = \frac{1}{2}M^2(T)\phi^2 - \frac{1}{3}\delta(T)\phi^3 + \frac{1}{4}\lambda\phi^4$$



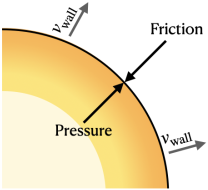


Credits: I. Stomberg



Hydrodynamics of first-order phase transitions[2]

- Broken-phase bubbles are nucleated and expand
- Friction from particles yield a terminal velocity ξ_w of the bubbles
- The bubble can run away when the friction is not enough to stop the bubble's acceleration



Boltzmann equation:

$$\left(p^\mu \nabla_\mu + \frac{1}{2}\partial_\mu m_X^2 \partial_{p_\mu}\right) f_X \, \delta^2(p^2 - m^2) = -C[f_X]$$

$$\Rightarrow \nabla_\mu T^{\mu\nu}_{\rm fluid} = -\partial^\nu \phi \sum_X \frac{{\rm d}m_X}{{\rm d}\phi} \int \frac{{\rm d}^3\vec{p}}{(2\pi)^3} \frac{1}{2E_X} f_X(\vec{x}, \vec{p})$$

$$f_{\rm eq} \to -\frac{\partial V_T}{\partial \phi}\partial^\nu \phi$$

$$\delta f \to -\eta u^\mu \partial_\mu \phi \partial^\nu \phi$$

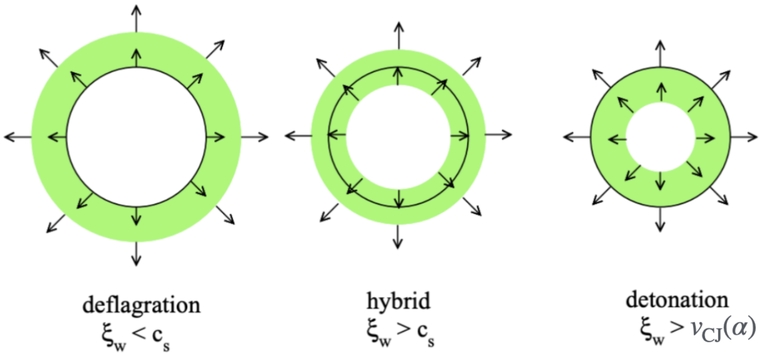
$$\nabla_\mu T^{\mu\nu}_\phi = \nabla_\mu (T^{\mu\nu}_\phi + T^{\mu\nu}_{\rm fluid}) = 0 \quad \Rightarrow \quad \begin{aligned} \nabla_\mu T^{\mu\nu}_\phi &= \frac{\partial V_T}{\partial \phi}\partial^\nu \phi + \eta u^\mu \partial_\mu \phi \partial^\nu \phi \\ \nabla_\mu T^{\mu\nu}_{\rm fluid} &= -\frac{\partial V_T}{\partial \phi}\partial^\nu \phi - \eta u^\mu \partial_\mu \phi \partial^\nu \phi \end{aligned}$$

[2] Espinosa, Konstandin, No, Servant, *JCAP* **06** (2010) 028.

Figueroa, Marschall, Midiri, ARP, in preparation

Hydrodynamics of first-order phase transitions[3]

- Depending on the wall velocity ξ_w and the strength of the phase transition $\alpha = \epsilon/\rho_{\rm rad}$, we can have 3 types of solution: *subsonic deflagrations, supersonic deflagrations (hybrids) or detonations.*



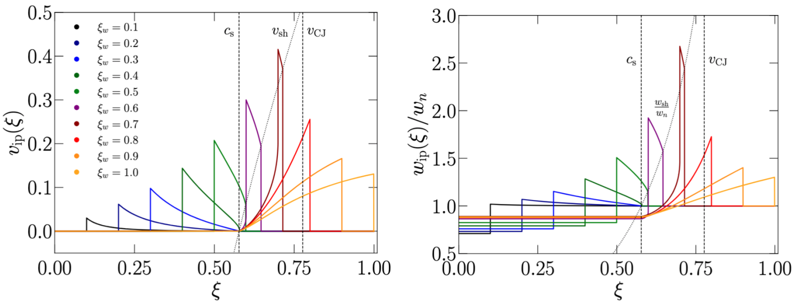
[3] Espinosa, Konstandin, No, Servant, *JCAP* **06** (2010) 028.

These calculations are available on CosmoGW,[4] using the bag EoS:

`pip install cosmoGW`

Bag EoS: $p_+ = aT^4 - \epsilon, \quad p_- = aT^4$

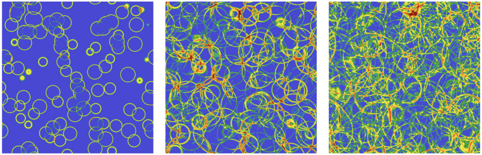
$\rho_+ = 3aT^4 + \epsilon, \quad \rho_- = 3aT^4$



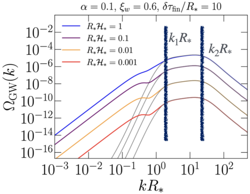
[4] https://github.com/CosmoGW/CosmoGW

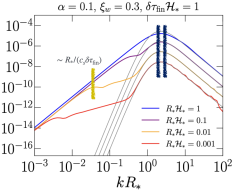
GWs from sound waves

- Numerical simulations of the scalar + fluid system performed by the Sussex/Helsinki group via an effective friction term indicate sound-wave regime to dominate for weak/intermediate phase transitions.[5]



- Sound-shell model.[6] Three scales are found to determine the GW spectrum: $k_1 = R_*$, $k_2 = \Delta R_*$ (sound-shell thickness), and $\delta\tau_{\rm fin}$ (sound-wave regime duration).





$$\Omega_{\rm GW}(k) = 3\,\tilde\Omega_{\rm GW}\,K^2\,\Upsilon(\tau_{\rm fin})\,(H_*R_*)\,S(kR_*)$$

$$\Upsilon(\tau_{\rm fin}) = \frac{H_*\tau_{\rm fin}}{1 + H_*\tau_{\rm fin}}$$

$$S(kR_*) = \mathcal{N}\left(\frac{k}{k_1}\right)^{n_1}\left[1 + \left(\frac{k}{k_1}\right)^{a_1}\right]^{\frac{-n_1+n_2}{a_1}}\left[1 + \left(\frac{k}{k_2}\right)^{a_2}\right]^{\frac{-n_2+n_3}{a_2}}$$

[5] Hindmarsh *et al.*, 2013, 2015, 2017, Cutting *et al.*, 2019, Correia *et al.*, 2025.

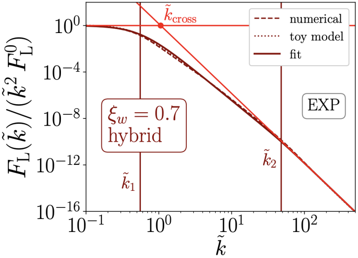
[6] Hindmarsh, 2016, Hindmarsh, Hijazi, 2019, ARP, Procacci, Caprini, 2023.

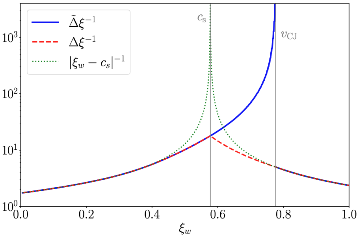
Scales and intermediate slope in the velocity spectrum[7]

- Identified two peaks directly from the velocity spectrum: $\tilde{k}_1$ and $\tilde{k}_2$, determined by positions of discontinuities in the 1D profiles, and by nucleation history.

$$\xi_1 \equiv \frac{2\pi}{z_1} \simeq \begin{cases} \frac{4}{3}\,(\xi_{\rm f} + \xi_w) & \text{for deflagrations}\,, \\ \frac{5}{3}\,(\xi_w + c_{\rm s}) & \text{for detonations}\,. \end{cases} \qquad \xi_2 \equiv \frac{2\pi}{z_2} \simeq \begin{cases} 2\,(\xi_{\rm f} - \xi_w) & \text{for deflagrations}\,, \\ \frac{4}{3}\,(\xi_w - c_{\rm s}) & \text{for detonations}\,. \end{cases}$$

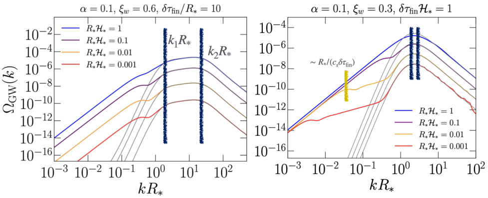
$$\frac{\tilde{k}_{1,2}^{\rm exp}}{z_{1,2}} \simeq \{0.17, 0.21\}\,, \qquad \frac{\tilde{k}_{1,2}^{\rm sim}}{z_{1,2}} \simeq \{0.38, 0.25\}\,.$$





7 Midiri, ARP *et al.*, arXiv:2604.02240 (2026).

Scales and intermediate slope **[Preliminary]**[8]



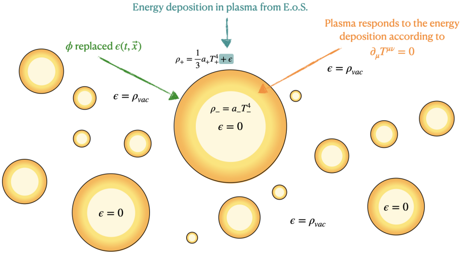
$$k_{1,2}^{\rm GW}/\beta \simeq 1.2\,\tilde{k}_{1,2},$$
$$\beta R_* = (8\pi)^{1/3}\xi_w$$

[8] Caprini, Midiri, ARP, in preparation.

GWs from sound waves: Higgsless simulations[10]

- Difficulty on simulations is due to the different scales of the scalar field ϕ and the fluid shell, so one can consider a nucleation history and set the pressure and energy density by knowing the value of ϵ and setting it during the simulation.
- Effect of bubble collisions on GWs is subdominant when sound waves are produced, so one can ignore the scalar field.
- Prescribes a terminal wall velocity, so slow down of the bubbles found, e.g., for strong deflagrations cannot be modeled.[9]
- Nucleation history is produced from an exponential probability distribution

$$P(t) \propto \exp\left[\beta(t - t_*)\right].$$



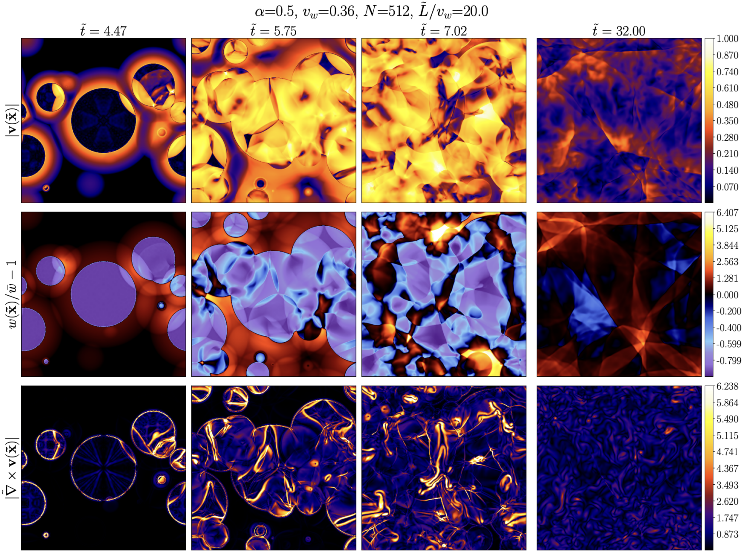
Credit: I. Stomberg

[9] Cutting et al., 2019; Correia et al., 2025.

[10] Jinno *et al.* JCAP 02 (2023) 011, 2209.04369,

ARP, Stomberg et al., JHEP **07** (2025) 217, arXiv:2409.03651.

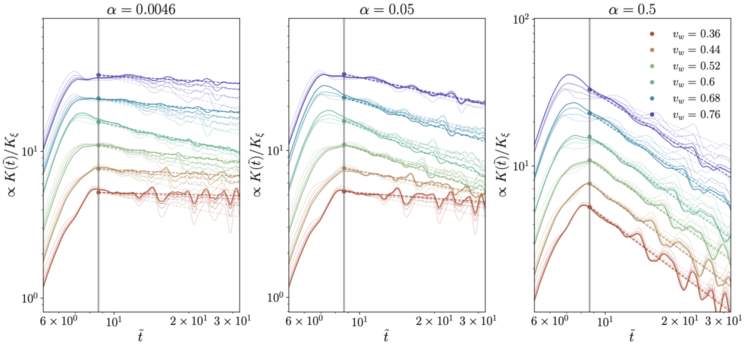
Higgsless simulations of strong PTs[11]



[11] ARP, Stomberg et al., JHEP **07** (2025) 217, arXiv:2409.03651.

Higgsless simulations (results)[12]

- Kinetic energy decay is observed in the simulations.
- For weak and strong PTs, increasing discretization enhances the decay.
- Potential indication of the development of non-linearities (turbulence).



[12] ARP, Stomberg et al., JHEP **07** (2025) 217, arXiv:2409.03651.

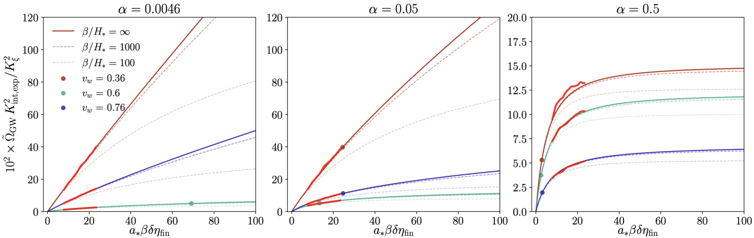
Higgsless simulations (results)[13]

- In the literature, the GW spectrum from sound waves is usually assumed to be

$$\Omega_{\rm GW}(f) = 3\,\tilde\Omega_{\rm GW}\, K^2\, \Upsilon(\tau_{\rm sw})\,(H_* R_*)\, S(f\, R_*)$$

- The linear growth, which only appears when expansion is neglected, is modified when the decay of the source is significant (e.g., due to the development of non-linearities).
- Extended model to proposed locally stationary UETC

$$\Omega_{\rm GW}(f) = 3\,\tilde\Omega_{\rm GW}\, \mathbf{K}^2_{\rm int,exp}\,(H_* R_*)\, S(f\, R_*)$$



13 ARP, Stomberg *et al.*, JHEP **07** (2025) 217, arXiv:2409.03651.

Higgsless simulations (results)[10]

- In the literature, the GW spectrum from sound waves is usually assumed to be

$$\Omega_{\rm GW}(f) = 3\,\tilde\Omega_{\rm GW}\,K^2\,\Upsilon(\tau_{\rm sw})\,(H_* R_*)\,S(f R_*$$

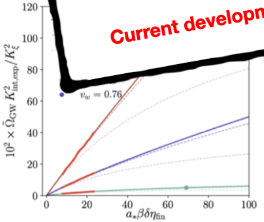
- The linear growth, which … is modified when the … ment of
- … Dahl *et*

Template for sound-shell model and template for non-linear sources based on Higgsless simulations available using public code CosmoGW (https://github.com/cosmoGW/cosmoGW)

proceedings on arxiv with I. Stomberg (2508.04263) and tutorial available on CosmoGW documentation

For use: pip install cosmoGW

Current developments undergoing to extend CosmoGW to other sources



$\alpha = 0.5$

17.5

Gravitational wave spectra for cosmological phase transitions with non-linear decay of the fluid motion

Isak Stomberg[a] and Alberto Roper Pol[b]

[a] *IFIC, Universitat de València-CSIC, C/ Catedràtico José Beltràn 2, E-46980, Paterna, Spain*
[b] *Département de Physique Théorique, Université de Genève, CH-1211 Genève, Switzerland*

[10] ARP, Stomberg *et al.*, JHEP, arXiv:2409.0

CosmoGW 1.0 and manual (2606.xxxx coming soon)

Classical and Quantum Gravity

CONTIBUTION TO "FOCUS ON THE LISA MISSION AT THE TIME OF ADOPTION"

CosmoGW: A public library to study cosmological backgrounds of gravitational waves. Part I — Phase transitions: Sound waves and MHD turbulence

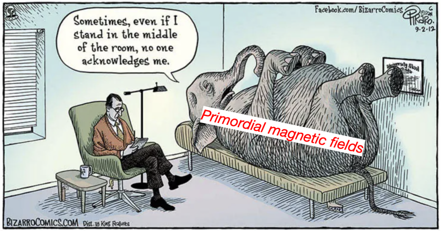
A. Roper Pol[1,*], M. Gurgenidze[2,3], A. S. Midiri[1], M. Salomé[1,4] and I. Stomberg[5]

Documentation available on Read the Docs, source code on GitHub.

- Routines to generate GW backgrounds from sound waves and turbulence associated to phase transitions
- Reading routines from numerical simulations (e.g., Pencil Code, Cosmo-Lattice)
- Cosmology calculations
- Interferometry, detector sensitivities

Primordial magnetic fields

- Magnetic fields can either be produced at or present during cosmological phase transitions.
- The magnetic fields are strongly coupled to the primordial plasma and effectively produce vortical motion, inevitably leading to the development of MHD turbulence.[14]
- Present magnetic fields can be amplified by primordial turbulence via dynamo.[15]



[14] J. Ahonen and K. Enqvist, *Phys. Lett. B* **382**, 40 (1996).
[15] A. Brandenburg *et al.* (incl. ARP), *Phys. Rev. Fluids* **4**, 024608 (2019).

Generation of primordial magnetic fields during inflation

- Magnetic fields could be amplified from quantum fluctuations during inflation.
- For inflationary magnetogenesis to be viable, conformal invariance needs to be broken,[16] with $\mathcal{L} \sim f(\phi)F_{\mu\nu}F^{\mu\nu}$ or $g(\phi)F_{\mu\nu}\tilde{F}^{\mu\nu}$. Otherwise, magnetic field perturbations would decay with the Universe expansion.
- After reheating, magnetic fields are strongly coupled to the primordial plasma and effectively produce vortical motion, inevitably leading to the development of MHD turbulence.[17]
- Alternatively, small seed magnetic fields can be amplified by primordial turbulence induced by phase transitions, e.g., via dynamo.[18]

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[16] Turner & Widrow (1988), Ratra (1992), Gasperini *et al.* (1995).

[17] J. Ahonen and K. Enqvist, *Phys. Lett. B* **382**, 40 (1996).

[18] A. Brandenburg *et al.* (incl. ARP), *Phys. Rev. Fluids* **4**, 024608 (2019).

Generation of primordial magnetic fields during phase transitions

- Parity-violating processes during the electroweak phase transition ($T \sim 100$ GeV) are predicted by SM extensions that account for baryogenesis and can produce helical magnetic fields through sphaleron decay or B+L anomalies.[19]

$$\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A} - i\frac{2\sin\theta_w}{gv^2}\boldsymbol{\nabla}\Phi^\dagger \times \boldsymbol{\nabla}\Phi$$

- Also after inflation, axion fields can amplify and produce magnetic fields.[20] For example, the QCD axion could oscillate and produce magnetic fields around the QCD scale ($T \sim 100$ MeV).[21]

$$\mathcal{L} \supset \frac{\phi}{f} F_{\mu\nu}\tilde{F}^{\mu\nu}$$

[19] T. Vachaspati, *Phys. Rev. B* **265**, 258 (1991), T. Vachaspati, *Phys. Rev. Lett.* **87**, 251302 (2001), J. M. Cornwall, *Phys. Rev. D* **56**, 6146 (1997).

[20] M. M. Forbes and A. R. Zhitnitsky, *Phys. Rev. Lett.* **85**, 5268 (2000).

[21] Miniati, Gregori, Reville & Sarkar (2018).

Generation of primordial magnetic fields from chiral anomalies

- Chiral magnetic effect in the early Universe at large temperatures $T > 80$ TeV can lead to the production of hypermagnetic fields.[22]
- Proposed chiral magnetic effect relevant at smaller scales[23] leading to a chiral plasma instability and development of magnetic fields that undergo inverse cascading.[24]
- Chiral magnetic effect also present for zero chiral chemical potential from local inhomogeneities.[25]
- Chiral magnetic effect can be activated from an active source of chiral chemical potential (e.g. associated to baryogenesis as the electroweak scale)[26]

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[22] M. Joyce and M. E. Shaposhnikov, *PRL* **79**, 1193 (1997)

[23] A. Boyarsky *et al.* *PRL* **108**, 031301 (2012).

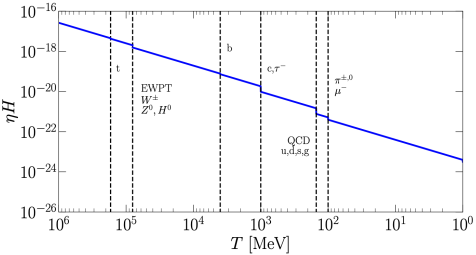
[24] I. Rogachevskii *et al.* *Astrophys. J. Lett* **845** (2017) L21, J. Schober *et al.* *Astrophy. J.* **858**, 124 (2018).

[25] J. Schober *et al.* *PRL* **132**, 6 (2024)

[26] M. Gurgenidze *et al.* (incl. ARP), arXiv:2512.09177 (2025).

MHD turbulence (large conductivity)

- High-conductivity (low magnetic diffusivity $\eta = 1/\sigma$) of the early universe leads to a high-coupling between magnetic and velocity fields.[27]



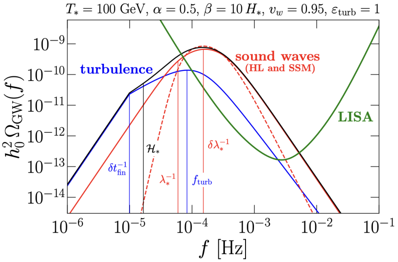
[27] ARP, Midiri, JCAP, 2026, arXiv:2501.05732.

GW sources in the early Universe

- Magnetohydrodynamic (MHD) sources of GWs:
  - Sound waves generated from first-order phase transitions.
  - (M)HD turbulence from first-order phase transitions.
  - Primordial magnetic fields.

- Other sources of GWs include
  - Bubble collisions.
  - Cosmic strings.
  - Primordial black holes.
  - Inflation.

ARP *et al.*, 2307.10744, 2308.12943



Primordial magnetic fields and GWs from MHD turbulence

- During the radiation-dominated era, after they are produced, magnetic fields will decay following MHD turbulence.
- Direct numerical simulations using the PENCIL CODE[28] to solve:
  1. Relativistic MHD equations adapted for radiation-dominated era (after electroweak symmetry is broken).
  2. Gravitational waves equation.
- In general, large-scale simulations are necessary to solve the MHD nonlinearities (e.g., unequal-time correlators UETC and non-Gaussianities, which require simplifying assumptions in analytical studies).

[28] Pencil Code Collaboration, JOSS **6**, 2807 (2020), `https://github.com/pencil-code/`

MHD description

Right after the electroweak phase transition we can model the plasma using continuum MHD.

- Charge-neutral, electrically conducting fluid.
- Relativistic magnetohydrodynamic (MHD) equations.
- Radiation-dominated fluid

$$p = \rho/3,$$

  i.e., $c_s^2 = 1/3$ (ultrarelativistic EoS).
- Friedmann–Lemaître–Robertson–Walker metric

$$g_{\mu\nu} = a^2 \eta_{\mu\nu}$$

Conservation laws for MHD turbulence

$$\nabla_\nu T^{\mu\nu} = 0, \quad \nabla_\nu F^{\mu\nu} = -J^\mu, \quad \nabla_\nu \mathcal{F}^{\mu\nu} = 0$$

In the limit of subrelativistic bulk flow:

$$\gamma^2 \sim 1 + (v/c)^2 + \mathcal{O}(v/c)^4$$

Relativistic MHD equations are reduced to[29]

$$\frac{\partial \ln \rho}{\partial t} = -\frac{4}{3}\left(\nabla \cdot \boldsymbol{u} + \boldsymbol{u} \cdot \nabla \ln \rho\right) + \frac{1}{\rho}\left[\boldsymbol{u} \cdot (\boldsymbol{J} \times \boldsymbol{B}) + \eta \boldsymbol{J}^2\right],$$

$$\frac{D\boldsymbol{u}}{Dt} = \frac{\boldsymbol{u}}{3}\left(\nabla \cdot \boldsymbol{u} + \boldsymbol{u} \cdot \nabla \ln \rho\right) - \frac{\boldsymbol{u}}{\rho}\left[\boldsymbol{u} \cdot (\boldsymbol{J} \times \boldsymbol{B}) + \eta J^2\right]$$

$$- \frac{1}{4}\nabla \ln \rho + \frac{3}{4\rho}\boldsymbol{J} \times \boldsymbol{B} + \frac{2}{\rho}\nabla \cdot (\rho\nu \boldsymbol{S}),$$

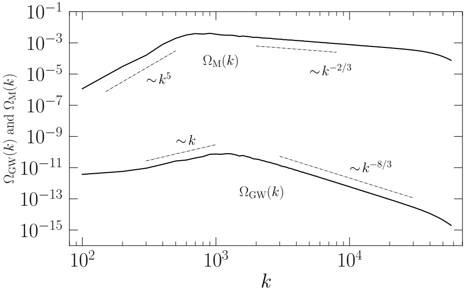
$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{u} \times \boldsymbol{B} - \eta \boldsymbol{J}), \quad \boldsymbol{J} = \nabla \times \boldsymbol{B},$$

for a flat expanding universe with comoving and normalized $p = a^4 p_{\rm phys}, \rho = a^4 \rho_{\rm phys}, B_i = a^2 B_{i,{\rm phys}}, u_i$, and conformal time t $({\rm d}t = a{\rm d}t_c)$.

[29] A. Brandenburg, *et al.*, *Phys. Rev. D* **54**, 1291 (1996).
ARP, Midiri, *Relativistic Magnetohydrodynamics in the early Universe*, accepted in JCAP, arXiv:2501.05732 (2025).

Numerical results for decaying MHD turbulence[30]

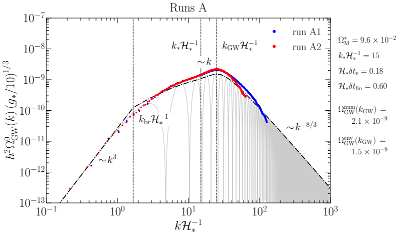
$1152^3, k_* = 2\pi \times 100, \Omega_{\rm M} \sim 10^{-2}, \sigma_{\rm M} = 1$

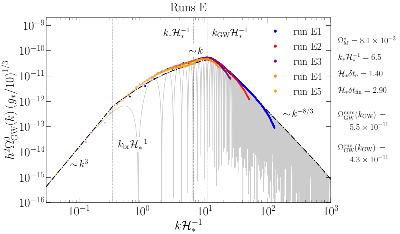


- **Characteristic k scaling in the subinertial range for the GW spectrum.**
- k^2 expected at scales $k < k_*$ and k^3 at $k < H_*$ according to the "top-hat" model (Caprini *et al.*, 2020).

[30] ARP *et al.*, *Phys. Rev. D* **102**, 083512 (2020).

Numerical results for decaying MHD turbulence[31]





run	$\Omega_{\rm M}^*$	$k_*\mathcal{H}_*^{-1}$	$\mathcal{H}_*\delta t_{\rm e}$	$\mathcal{H}_*\delta t_{\rm fin}$	$\Omega_{\rm GW}^{\rm num}(k_{\rm GW})$	$[\Omega_{\rm GW}^{\rm env}/\Omega_{\rm GW}^{\rm num}](k_{\rm GW})$	n	$\mathcal{H}_*L$	$\mathcal{H}_*t_{\rm end}$	$\mathcal{H}_*\eta$
A1	9.6×10^{-2}	15	0.176	0.60	2.1×10^{-9}	1.357	768	6π	9	10^{-7}
A2	–	–	–	–	–	–	768	12π	9	10^{-6}
E1	8.1×10^{-3}	6.5	1.398	2.90	5.5×10^{-11}	1.184	512	4π	8	10^{-7}
E2	–	–	–	–	–	–	512	10π	18	10^{-7}
E3	–	–	–	–	–	–	512	20π	61	10^{-7}
E4	–	–	–	–	–	–	512	30π	114	10^{-7}
E5	–	–	–	–	–	–	512	60π	234	10^{-7}

[31] ARP *et al.*, *Phys. Rev. D* **105**, 123502 (2022).

Analytical model for GWs from decaying turbulence

- Assumption: magnetic or velocity field evolution $\delta t_{\rm e} \sim 1/(u_* k_*)$ is slow compared to the GW dynamics ($\delta t_{\rm GW} \sim 1/k$) at all $k \gtrsim u_* k_*$.
- We can derive an analytical expression for nonhelical fields of the envelope of the oscillations[32] of $\Omega_{\rm GW}(k)$.

$$\Omega_{\rm GW}(k, t_{\rm fin}) \approx 3 \left(\frac{k}{k_*}\right)^3 {\Omega_{\rm M}^*}^2 \frac{\mathcal{C}(\alpha)}{\mathcal{A}^2(\alpha)}\, p_\Pi\left(\frac{k}{k_*}\right) \times \begin{cases} \ln^2[1 + \mathcal{H}_* \delta t_{\rm fin}] & \text{if } k\, \delta t_{\rm fin} < 1, \\ \ln^2[1 + (k/\mathcal{H}_*)^{-1}] & \text{if } k\, \delta t_{\rm fin} \geq 1. \end{cases}$$

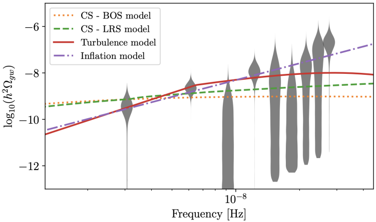
- p_Π is the anisotropic stress spectrum and depends on spectral shape, can be approximated for a von Kárman spectrum as[33]

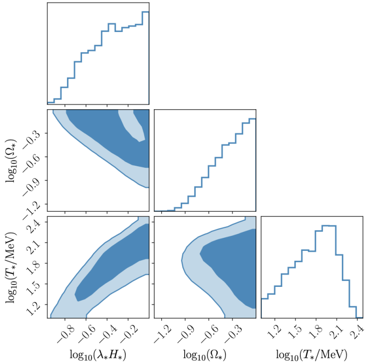
$$p_\Pi(k/k_*) \simeq \left[1 + \left(\frac{k}{2.2 k_*}\right)^{2.15}\right]^{-11/(3\times 2.15)}$$

[32] ARP *et al.*, *Phys. Rev. D* **105**, 123502 (2022); ARP, Midiri, Salomé, Caprini, in preparation.

[33] ARP *et al.*, arXiv:2307.10744 (2023).

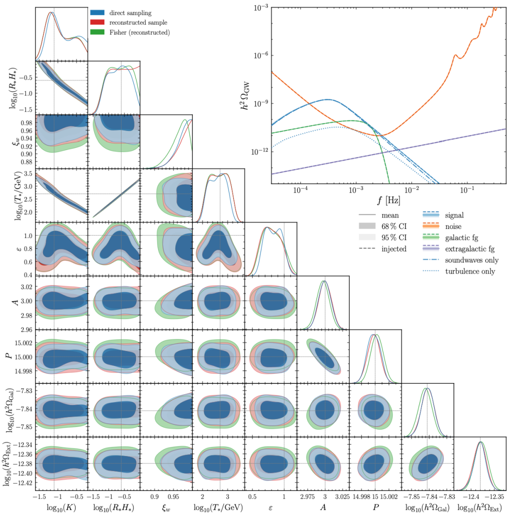
Primordial magnetic fields constraints with PTA[34]





[34] [EPTA collab.] (incl. ARP), arXiv:2306.16227 (2023).

Parameter reconstruction of GW signal for sound waves and turbulence from a PT with LISA[35]



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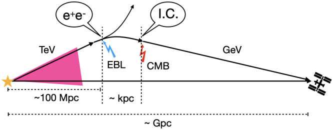
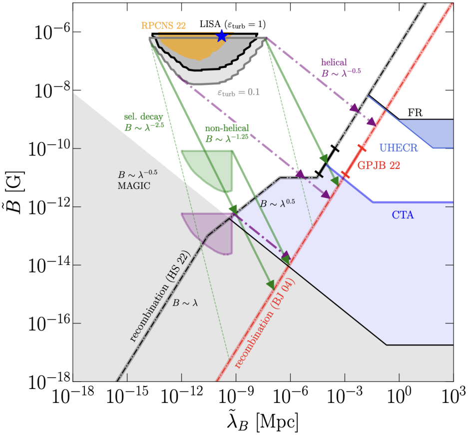
[35] [LISA CosWG] (incl. ARP), arXiv:2403.03723 (2024).

Multi-messenger studies of primordial magnetic fields[30]

- Primordial magnetic fields would evolve through the history of the universe up to the present time and could explain the lower bounds in cosmic voids derived by the Fermi collaboration.[31]



- Maximum amplitude of primordial magnetic fields is constrained by the big bang nucleosynthesis.[32]
- Additional constraints from CMB, Faraday Rotation, ultra-high energy cosmic rays (UHECR).



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[30] ARP *et al.*, arXiv:2307.10744 (2023).

[31] A. Neronov and I. Vovk, *Science* **328**, 73 (2010).

[32] V. F. Shvartsman, *Pisma Zh. Eksp. Teor. Fiz.* **9**, 315 (1969).

Conclusions 1/2

- **Velocity and magnetic fields** in the early universe can significantly **contribute to the cosmological GW background (GWB) via sound waves and (M)HD turbulence**.
- An observation of a GWB from the early Universe could **probe BSM physics** that are not accessible to electromagnetic observations and that can be complementary to collider experiments in some BSM theories.
- The GWB produced by **non-linear fluid (and magnetic field) dynamics** requires, in general, performing high-resolution **numerical simulations**, which can be done using open-source codes like PENCIL CODE or $\mathcal{C}$osmo$\mathcal{L}$attice.
- We expect the **dynamics of first-order phase transitions to be affected by gauge fields**, with its impact on the resulting GWB and on the fluid dynamics sourcing GWs being at the moment uncertain.

Conclusions 2/2

- **Magnetic fields are ubiquitous in nature at all scales,** from the smallest structures up to galaxies, cluster of galaxies, filaments, and potentially in cosmic voids of the LSS.
- **Primordial magnetic fields** produced in the early Universe **can be probed** by other **cosmological** (CMB) and **astrophysical** (γ-ray, radio observations at voids, filaments of the LSS) observations, providing us a multi-messenger approach to study early Universe physics.
- **To extract clean information** from a GWB observation in LISA, PTA, or ET, we will need to have **accurate modeling of the resulting GWB**, since the reconstruction of the signal is very challenging (multiple cosmo sources, astrophysical background, detectors noise).
- **Current state-of-the-art models provide good estimates but still require improvement** for extracting clean information of the underlying source. Python package CosmoGW available.

Thank You!

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github.com/AlbertoRoper/cosmoGW

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