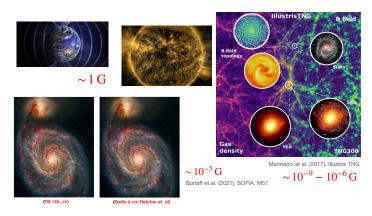
Primordial magnetic fields: origin, evolution, and connection to GW production in the early Universe

Pencil Code school (Oct. 23, 2025, CERN)

Our magnetized Universe

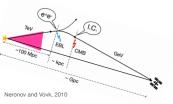


- Dense regions in our Universe are strongly magnetized with $B\sim 10\mu{\rm G}$ in galaxies and clusters ($\sim 10~{\rm kpc}$). Radio and far-infrared (FIR) observations.
- Filaments also contain magnetic fields with $B\sim$ 10 nG (LOFAR observations of Faraday rotation measurements).

Intergalactic magnetic fields in voids of the LSS



- γ-ray observations from distant blazars by Fermi/MAGIC collaboration show a removal of power at GeV, providing evidence for an intergalactic magnetic field.
- Lower bound of $B \sim 10^{-16}$ G at Mpc scales.



Relics from the early Universe?

- The observed intergalactic magnetic fields could have a primordial (from inflation or from a phase transition) or an astrophysical origin.
- Observations indicate a large volume filling factor in the voids and large correlation length scales (Mpc), favoring a primordial origin.
- Alternative possibilities to explain the blazar observations are:
 - Beam-plasma instabilities (suppressing the GeV emission) proposed¹, but seems to not be significant at the intergalactic scales.
 - Magnetized galactic outflows. Studied using cosmological simulations.
 However, it is difficult to reach large volume filling factors.²
 - Contribution from galactic dipoles,³ seems plausible but further studies are required.
- Primordial magnetic field present at recombination could alleviate the Hubble tension by reducing the sound horizon.⁴



¹Broderick et al. (2018).

²Ni et al. (2024), Tjemsland et al. (2024).

³Garg, Durrer & Schober (2025).

⁴ Jedamzik & Pogosian (2020).

Generation of primordial magnetic fields

- Magnetic fields could be amplified from quantum fluctuations during inflation or during cosmological phase transitions.
- For inflationary magnetogenesis to be viable, conformal invariance needs to be broken, with $\mathcal{L} \sim f(\phi) F_{\mu\nu} F^{\mu\nu}$ or $g(\phi) F_{\mu\nu} \tilde{F}^{\mu\nu}$. Otherwise, magnetic field perturbations would decay with expansion.
- After reheating, magnetic fields are strongly coupled to the primordial plasma and effectively produce vortical motion, inevitably leading to the development of MHD turbulence.⁶
- Alternatively, small seed magnetic fields can be amplified by primordial turbulence induced by phase transitions, e.g., via dynamo.⁷

⁵Turner & Widrow (1988), Ratra (1992), Gasperini et al. (1995).

⁶ J. Ahonen and K. Enqvist, *Phys. Lett. B* **382**, 40 (1996).

⁷ A. Brandenburg et al. (incl. ARP), Phys. Rev. Fluids 4, 024608 (2019): □ > ← (□) >

Generation of primordial magnetic fields

• Parity-violating processes during the electroweak phase transition ($T\sim 100~\text{GeV}$) are predicted by SM extensions that account for baryogenesis and can produce helical magnetic fields through sphaleron decay or B+L anomalies.⁸

$$\mathbf{B} = \mathbf{\nabla} \times \mathbf{A} - i \frac{2 \sin \theta_w}{g v^2} \mathbf{\nabla} \Phi^{\dagger} \times \mathbf{\nabla} \Phi$$

• Also after inflation, axion fields can amplify and produce magnetic fields. For example, the QCD axion could oscillate and produce magnetic fields around the QCD scale ($T\sim 100~\text{MeV}$). 10

$$\mathcal{L}\supsetrac{\phi}{f}F_{\mu
u} ilde{F}^{\mu
u}$$



⁸T. Vachaspati, Phys. Rev. B **265**, 258 (1991), T. Vachaspati, Phys. Rev. Lett. **87**, 251302 (2001), J. M. Cornwall, Phys. Rev. D **56**, 6146 (1997).

⁹M. M. Forbes and A. R. Zhitnitsky, *Phys. Rev. Lett.* **85**, 5268 (2000).

Miniati, Gregori, Reville & Sarkar (2018).

Generation of primordial magnetic fields

- Inhomogeneities in the Higgs field in low-scale electroweak hybrid inflation.¹¹
- Magnetic fields from inflation can be present and amplified during phase transitions (non-helical¹² and helical¹³).
- Chiral magnetic effect.¹⁴



¹¹ M. Joyce and M. E. Shaposhnikov, *Phys. Rev. Lett.* **79**, 1193 (1997), J. García-Bellido *et al.*, *Phys. Rev. D* **60**, 123504 (1999).

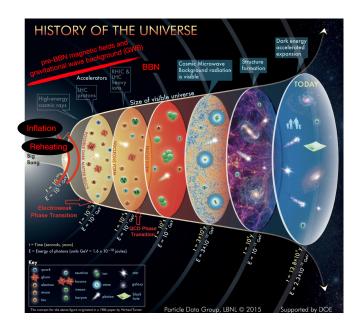
¹² M. S. Turner and L. M. Widrow, Phys. Rev. D 37, 2743 (1988).

¹³M. Giovannini, *Phys. Rev. D* **58**, 124027 (1998).

¹⁴ M. Joyce and M. E. Shaposhnikov, PRL 79, 1193 (1997).

GWs from the early Universe

- Gravity is the weakest fundamental force. Hence, GWs are difficult to detect but they propagate freely carrying clean information of the source.
- Primordial magnetic fields and MHD turbulence in the primordial plasma can produce a gravitational wave background potentially observable with LISA, PTA, or other GW experiments.
- Observations of gravitational wave backgrounds and primordial magnetic fields can be combined for multi-messenger studies of the primordial Universe.
- GWs from the early Universe have the potential to provide us with direct information on early universe physics that is not accessible via electromagnetic observations, complementary to collider experiments.



Probing the early Universe with GWs Cosmological (pre-recombination) GW background

 Why background? Individual sources are not resoluble, superposition of single events occurring in the whole Universe.

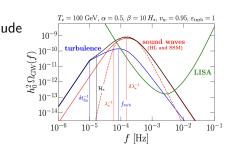
$$f_* \simeq 1.64 \times 10^{-3} \frac{100}{R_* \mathcal{H}_*} \frac{T_*}{100 \, {\rm GeV}} \, {\rm Hz}$$

- Phase transitions
 - Ground-based detectors (LVK, ET, CE) frequencies are 10–1000 Hz Peccei-Quinn, B-L, left-right symmetries $\sim 10^7, 10^8$ GeV.
 - Space-based detectors (LISA) frequencies are 10^{-5} – 10^{-2} Hz Electroweak phase transition ~ 100 GeV
 - Pulsar Timing Array (PTA) frequencies are 10^{-9} – 10^{-7} Hz Quark confinement (QCD) phase transition ~ 100 MeV

GW sources in the early Universe

- Magnetohydrodynamic (MHD) sources of GWs:
 - Sound waves generated from first-order phase transitions.
 - (M)HD turbulence from first-order phase transitions.
 - Primordial magnetic fields.
- High-conductivity of the early universe leads to a high-coupling between magnetic and velocity fields.
- Other sources of GWs include
 - Bubble collisions.
 - Cosmic strings.
 - Primordial black holes.
 - Inflation.

ARP et al., 2307.10744, 2308.12943



Primordial magnetic fields and GWs from MHD turbulence

- During the radiation-dominated era, after they are produced, magnetic fields will decay following MHD turbulence.
- Direct numerical simulations using the Pencil Code¹⁵ to solve:
 - Relativistic MHD equations adapted for radiation-dominated era (after electroweak symmetry is broken).
 - ② Gravitational waves equation.
- In general, large-scale simulations are necessary to solve the MHD nonlinearities (e.g., unequal-time correlators UETC and non-Gaussianities, which require simplifying assumptions in analytical studies).
- Currently, CosmoLattice-MHD module is under-development (work with D. Figueroa, K. Marschall, A. Midiri).



Pencil Code Collaboration, JOSS 6, 2807 (2020), https://github.com/pencil-code/

MHD description

Right after the electroweak phase transition we can model the plasma using continuum MHD.

- Charge-neutral, electrically conducting fluid.
- Relativistic magnetohydrodynamic (MHD) equations.
- Radiation-dominated fluid

$$p = \rho c^2/3,$$

i.e., $c_s^2 = 1/3$ (ultrarelativistic EoS).

Friedmann-Lemaître-Robertson-Walker metric

$$g_{\mu\nu} = \text{diag}\{-1, a^2, a^2, a^2\}$$



Conservation laws for MHD turbulence

$$T^{\mu\nu}_{\ ;\nu} = 0, \quad F^{\mu\nu}_{\ ;\nu} = -J^{\mu}, \quad \tilde{F}^{\mu\nu}_{\ ;\nu} = 0$$

In the limit of subrelativistic bulk flow:

$$\gamma^2 \sim 1 + (v/c)^2 + \mathcal{O}(v/c)^4$$

Relativistic MHD equations are reduced to 16

$$\frac{\partial \ln \rho}{\partial t} = -\frac{4}{3} (\nabla \cdot \boldsymbol{u} + \boldsymbol{u} \cdot \nabla \ln \rho) + \frac{1}{\rho} [\boldsymbol{u} \cdot (\boldsymbol{J} \times \boldsymbol{B}) + \eta \boldsymbol{J}^{2}],$$

$$\frac{D\boldsymbol{u}}{Dt} = \frac{\mathbf{u}}{3} (\nabla \cdot \boldsymbol{u} + \boldsymbol{u} \cdot \nabla \ln \rho) - \frac{\boldsymbol{u}}{\rho} [\boldsymbol{u} \cdot (\boldsymbol{J} \times \boldsymbol{B}) + \eta \boldsymbol{J}^{2}]$$

$$-\frac{1}{4} \nabla \ln \rho + \frac{3}{4\rho} \boldsymbol{J} \times \boldsymbol{B} + \frac{2}{\rho} \nabla \cdot (\rho \nu \boldsymbol{S}),$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{u} \times \boldsymbol{B} - \eta \boldsymbol{J}), \quad \boldsymbol{J} = \nabla \times \boldsymbol{B},$$
(1)

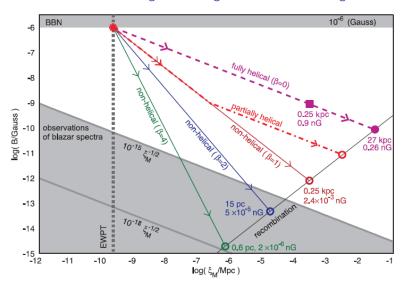
for a flat expanding universe with comoving and normalized

$$p = a^4 p_{\text{phys}}, \rho = a^4 \rho_{\text{phys}}, B_i = a^2 B_{i,\text{phys}}, u_i, \text{ and conformal time } t \text{ } (dt = a dt_c).$$

A. Brandenburg, et al., Phys. Rev. D 54, 1291 (1996).
ARP, Midiri, Relativistic Magnetohydrodynamics in the early Universe, arXiv:2501.05732 (2025).



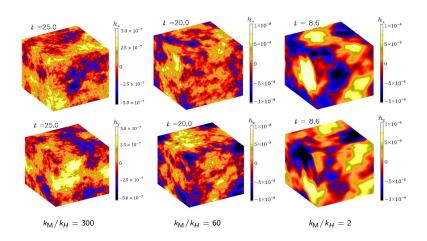
Evolution of magnetic strength and correlation length¹⁷



¹⁷ A. Brandenburg et al. (incl. ARP), Phys. Rev. D 96, 123528 (2017).



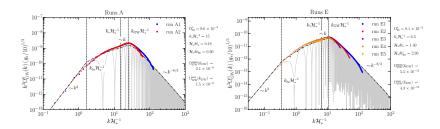
Numerical results for decaying MHD turbulence 18 $1152^3, \Omega_{\rm M} \sim 10^{-2}$





¹⁸ARP et al., Phys. Rev. D **102**, 083512 (2020).

Numerical results for decaying MHD turbulence¹⁹



run	Ω_{M}^{*}	$k_*\mathcal{H}_*^{-1}$	$\mathcal{H}_*\delta t_e$	$\mathcal{H}_*\delta t_{\mathrm{fin}}$	$\Omega_{\rm GW}^{\rm num}(k_{\rm GW})$	$[\Omega_{\rm GW}^{\rm env}/\Omega_{\rm GW}^{\rm num}](k_{\rm GW})$	n	\mathcal{H}_*L	$\mathcal{H}_*t_{\mathrm{end}}$	$\mathcal{H}_*\eta$
A1	9.6×10^{-2}	15	0.176	0.60	2.1×10^{-9}	1.357	768	6π	9	10^{-7}
A2	-	-	-	-	-	-	768	12π	9	10^{-6}
E1	8.1×10^{-3}	6.5	1.398	2.90	5.5×10^{-11}	1.184	512	4π	8	10^{-7}
E2	-	-	-	-	-	-	512	10π	18	10^{-7}
E3	-	-	-	-	-	-	512	20π	61	10^{-7}
E4	-	-	-	-	-	-	512	30π	114	10^{-7}
E5	_	_	_	_	_	_	512	60π	234	10^{-7}



¹⁹ARP et al., Phys. Rev. D **105**, 123502 (2022).

Analytical model for GWs from decaying turbulence

- Assumption: magnetic or velocity field evolution $\delta t_{\rm e} \sim 1/(u_* k_*)$ is slow compared to the GW dynamics $(\delta t_{\rm GW} \sim 1/k)$ at all $k \gtrsim u_* k_*$.
- We can derive an analytical expression for nonhelical fields of the envelope of the oscillations²⁰ of Ω_{GW}(k).

$$\Omega_{\rm GW}(k, t_{\rm fin}) \approx 3 \left(\frac{k}{k_*}\right)^3 \Omega_{\rm M}^{*2} \frac{\mathcal{C}(\alpha)}{\mathcal{A}^2(\alpha)} p_{\rm \Pi}\left(\frac{k}{k_*}\right)$$
$$\times \begin{cases} \ln^2[1 + \mathcal{H}_* \delta t_{\rm fin}] & \text{if } k \, \delta t_{\rm fin} < 1, \\ \ln^2[1 + (k/\mathcal{H}_*)^{-1}] & \text{if } k \, \delta t_{\rm fin} \ge 1. \end{cases}$$

• p_{Π} is the anisotropic stress spectrum and depends on spectral shape, can be approximated for a von Kárman spectrum as²¹

$$p_{\Pi}(k/k_*) \simeq \left[1 + \left(\frac{k}{2.2k_*}\right)^{2.15}\right]^{-11/(3\times2.15)}$$

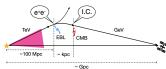


²⁰ARP et al., Phys. Rev. D **105**, 123502 (2022).

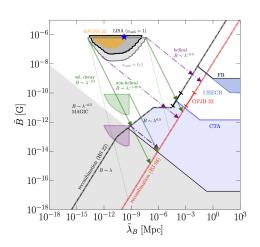
²¹ ARP et al., arXiv:2307.10744 (2023).

Multi-messenger studies of primordial magnetic fields³⁰

 Primordial magnetic fields would evolve through the history of the universe up to the present time and could explain the lower bounds in cosmic voids derived by the Fermi collaboration.³¹



- Maximum amplitude of primordial magnetic fields is constrained by the big bang nucleosynthesis.³²
- Additional constraints from CMB,
 Faraday Rotation, ultra-high energy cosmic rays (UHECR).



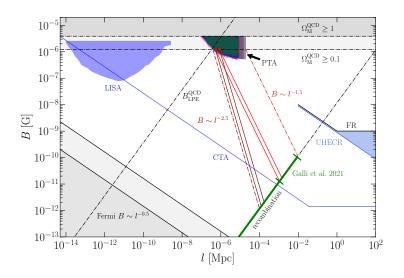


³⁰ARP *et al.*, arXiv:2307.10744 (2023).

³¹A. Neronov and I. Vovk, *Science* **328**, 73 (2010).

³²V. F. Shvartsman, *Pisma Zh. Eksp. Teor. Fiz.* **9**, 315 (1969).

Primordial magnetic field constraints with PTA²²



 $^{^{22}}$ ARP et al., Phys. Rev. D **105**, 123502 (2022).



Conclusions

- Magnetic fields are ubiquitous in nature at all scales, from the smallest structures up to galaxies, cluster of galaxies, filaments, and potentially in cosmic voids of the LSS.
- Observations from γ -rays indicate the presence of intergalactic magnetic fields that could have their origin in the early Universe and potentially alleviate the Hubble tension.
- Magnetic fields can be produced in the early Universe during the period of inflation or during a cosmological phase transition (EW or QCD), leaving at the same time an imprint on the gravitational wave background (GWB).
- The large conductivity during the radiation-dominated era implies the development of MHD turbulence and therefore, the primordial plasma is non-linearly stirred by the magnetic field.
- LISA, PTA, and next-generation ground-based detectors can be used to probe the origin of magnetic fields in the largest scales of our Universe, which is still an open question in cosmology.
- In summary, magnetic fields can be used as multi-messenger probes for early Universe physics combining astrophysical and cosmological observations, and can help us understanding high-energy physics, at scales inaccessible by other means.











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github.com/AlbertoRoper/cosmoGW cosmology.unige.ch/users/alberto-roper-pol







