# The Inflationary Era in Cosmology: Studying Axion Inflation with the Pencil Code


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Accelerating the Pencil Code

Pencil Code
School and User
meeting 2025

CERN

# Outline

- Introduction to hot Big Bang model
- Motivation to have an inflationary era
  - Horizon Problem
  - Flatness Problem
- Inflation : A solution to these problem
- A simple model of inflation
- Axion Inflation model
- Implementation of Axion-U(1) model in Pencil code
  - backreact_infl.f90 and disp_current.f90
  - klein_gorden.f90
- Other studies in the context of inflation using Pencil Code (axionSU2back.f90)

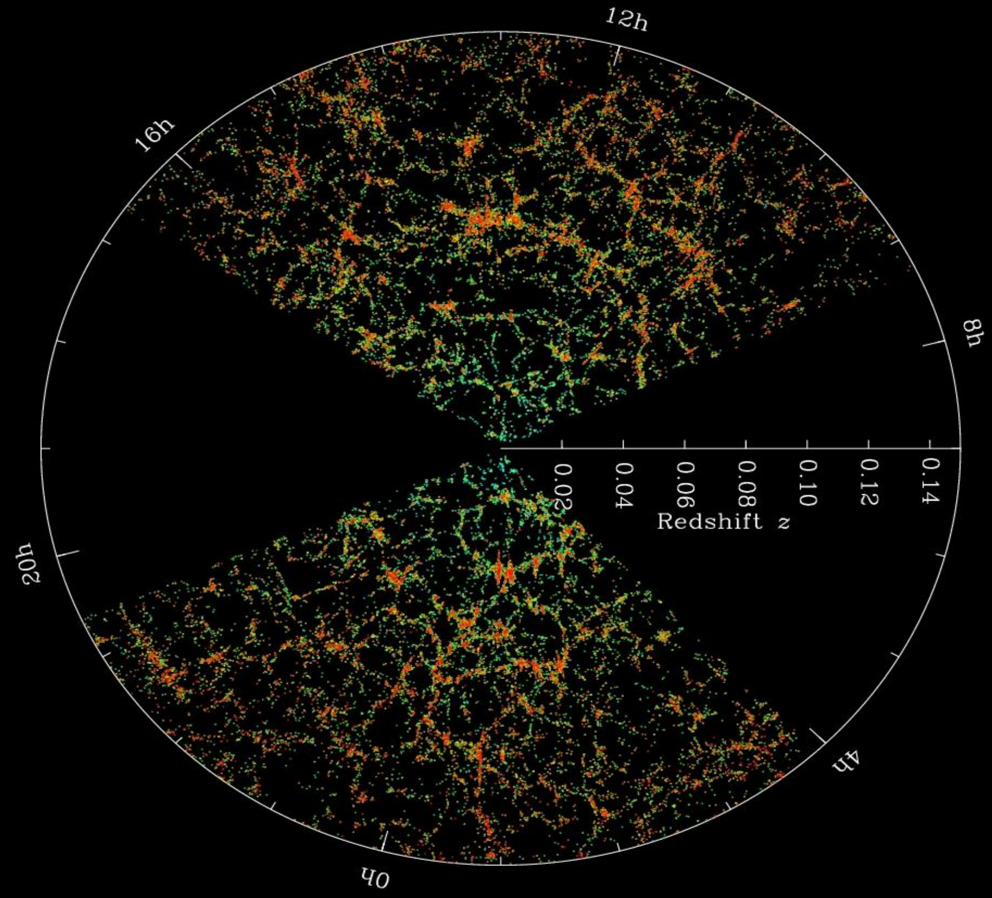
# Length scales

- Size of milky way ~ 20 Kpc ~$10^9$A.U. ~ $10^4$ size of solar system ~ 6 * $10^4$ light year

- Size of a galaxy cluster ~ 1Mpc

NASA.GOV

# Distribution of the galaxies




SDSS sky survey

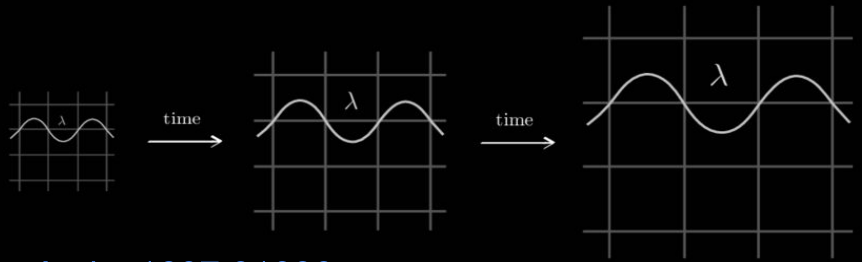
- Density contrast ~ $10^5$
- At around 100 Mpc Universe is homogeneous  Yadav et al 2005
- Universe is expanding with time

  Edwin Hubble 1929
- Horizon ~ 4000 Mpc
- Distance between two points

$$dx^2 + dy^2$$

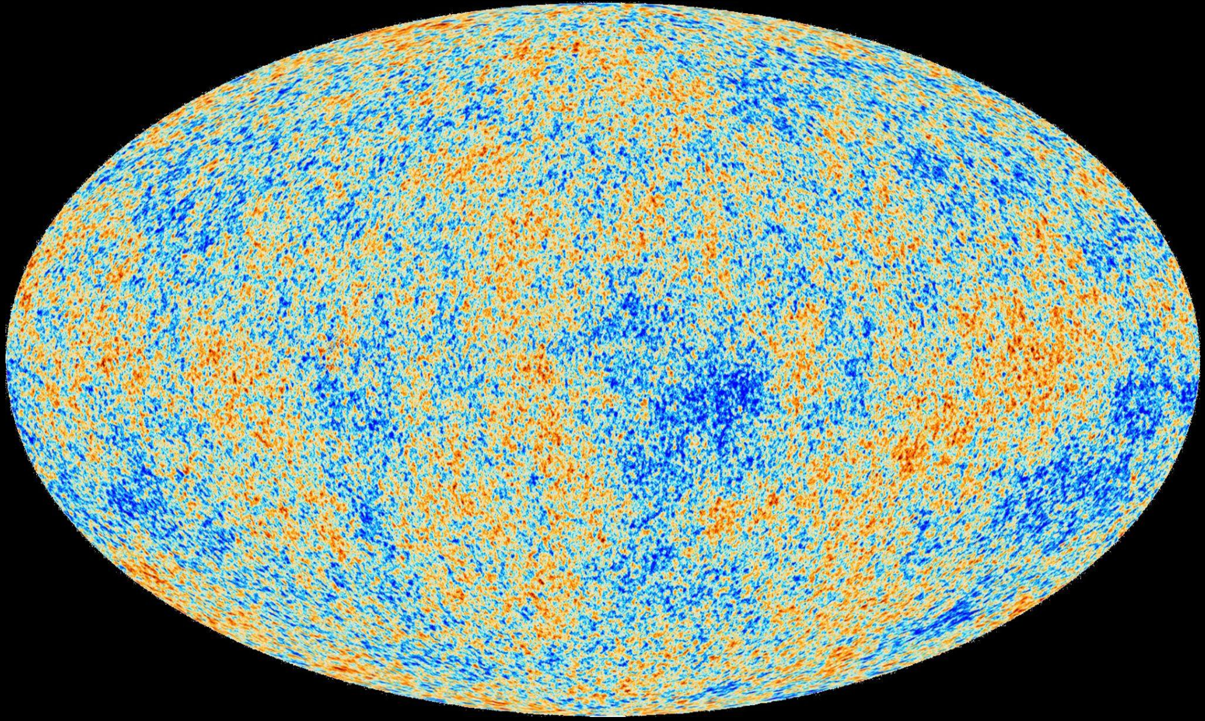
- In the case of expanding

$$a^2(t)(dx^2 + dy^2)$$




Arxiv: 1607.01030

# Cosmic Microwave Background Radiation (CMBR)



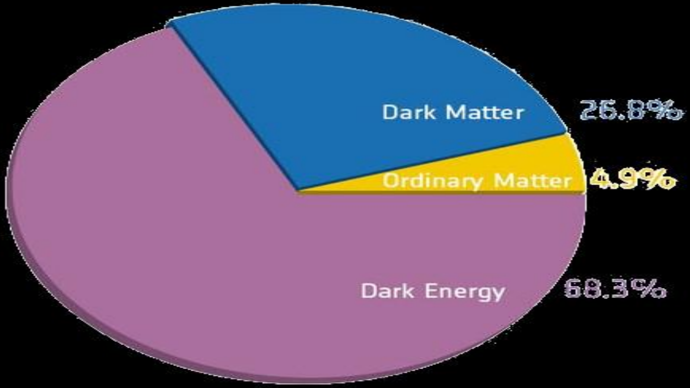
ESA: Planck mission

Temp. fluctuations ~ $10^{-5}$

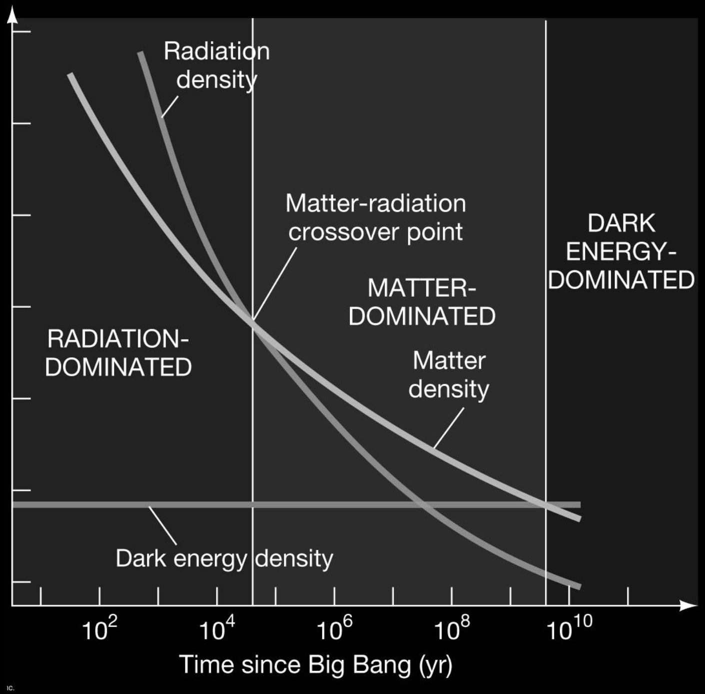
Density fluctuations ~ $10^{-5}$

# Cosmology

- At large scale, Universe is homogeneous and Isotropic
- Universe is expanding
- Matter density $\propto \frac{1}{a^3}$
- 
- Radiation density $\propto \frac{1}{a^4}$ and T $\propto \frac{1}{a}$




ESA: Planck's mission



# Hot Big Bang phase

- Explains cosmic expansion, nucleosynthesis, and CMB formation.
- Predicts a hot, dense early universe.
- However, leaves some questions unanswered:
  - Why is the universe so uniform on large scales? (Horizon Problem)
  - Why is the universe so flat today? (Flatness problem)

$$\begin{aligned} ds^2 &= dt^2 - a^2(t)(dx^2 + dy^2 + dz^2) \\ &= a^2(\eta)(d\eta^2 - (dx^2 + dy^2 + dz^2)) \end{aligned}$$

- Using Einstein equation for perfect fluid, one gets

$$\left(\frac{\dot{a}}{a}\right)^2 \equiv H^2 = \frac{8\pi G}{3}(\rho_r + \rho_m + \rho_\Lambda)$$

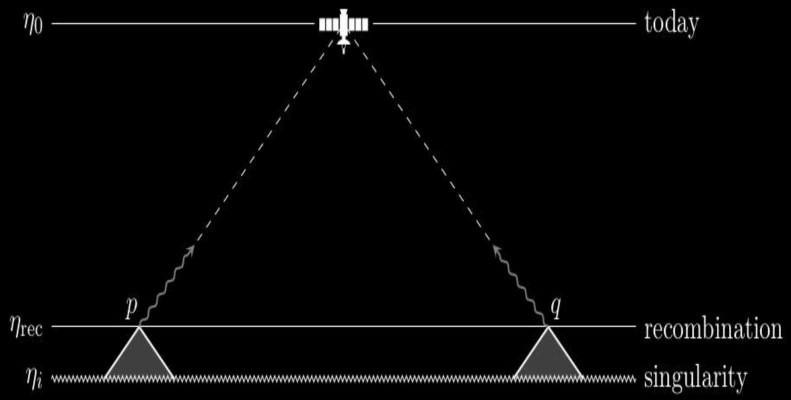
$$H^2 = \frac{8\pi G}{3}\left(\rho_{r,0}\left(\frac{a_0}{a}\right)^4 + \rho_{m,0}\left(\frac{a_0}{a}\right)^3 + \rho_{\Lambda,0}\right)$$

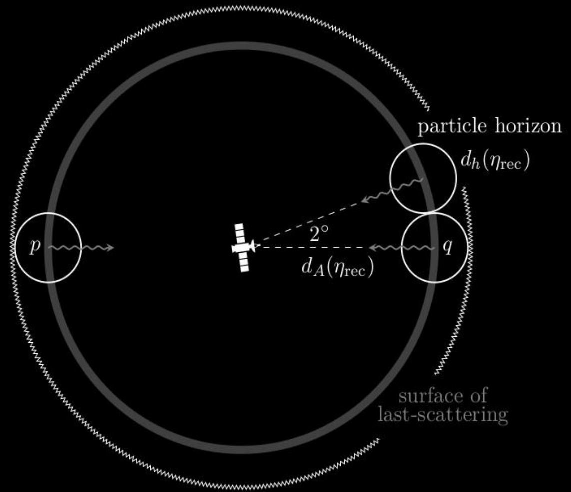
# Horizon Problem

- The CMB have approximately same temperature across the sky ($\Delta T/T \sim 10^{-5}$).
- Opposite regions of the sky were never in causal contact at recombination.
- Light couldn't have traveled between them since the Big Bang - to thermalize.
- Yet, they have the same temperature - why?

$$\text{For Photons, } ds = 0 \Rightarrow dx = d\eta = \frac{d\ln a}{aH}$$

$$\text{Distance travelled, } d_h = \int \frac{d\ln a}{aH}$$




Daniel Baumann, Cosmology, Cambridge University Press, 2022

# Flatness Problem

- Friedmann equation:

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho$$

$$H^2 = \frac{8\pi G}{3}\rho_c$$

$$\Omega_k = \frac{\rho - \rho_c}{\rho_c} = \frac{a_0 H_0^2}{a^2 H^2}\Omega_{k,0}$$

- Small deviation from flatness ($\Omega \equiv \frac{\rho}{\rho_c}$=1 ) grows with time in standard expansion.
- From CMB observations ($|\Omega_{k,0}| < 0.005$)
- Implies extreme fine-tuning of initial conditions.

$$\left(\frac{\dot{a}}{a}\right)^2 \equiv H^2 = \frac{8\pi G}{3}(\rho_r + \rho_m + \rho_\Lambda)$$

$$H^2 = \frac{8\pi G}{3}\left(\rho_{r,0}\left(\frac{a_0}{a}\right)^4 + \rho_{m,0}\left(\frac{a_0}{a}\right)^3 + \rho_{\Lambda,0}\right)$$

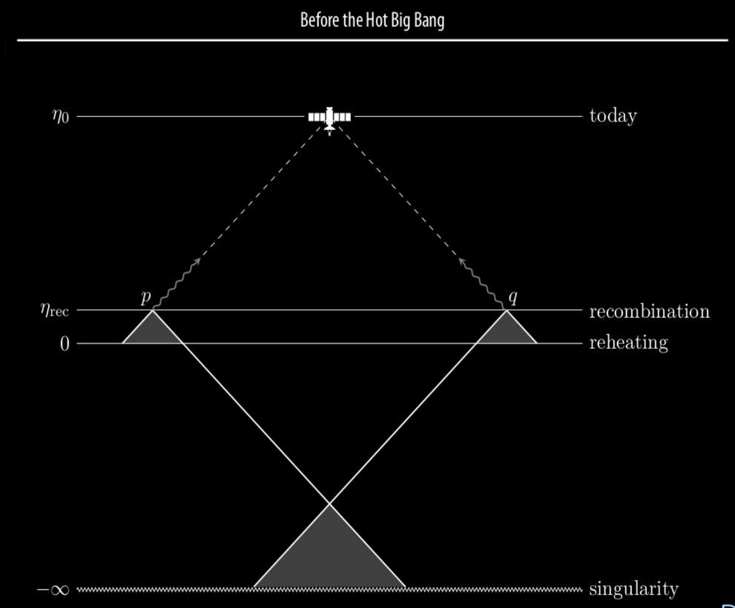
# Solution to these problems: Inflationary Era

Decreasing Hubble radius can resolve the Horizon and Flatness Problem

$$\frac{d}{dt}(aH)^{-1} < 0 \Rightarrow \ddot{a} > 0 \quad \text{(accelerated expansion)}$$

$$\frac{\ddot{a}}{a} = \frac{-4\pi G}{3}(\rho + 3p)$$

$$\text{accelerated expansion implies} \quad \rho + 3p < 0 \quad \Rightarrow p < -\rho/3$$

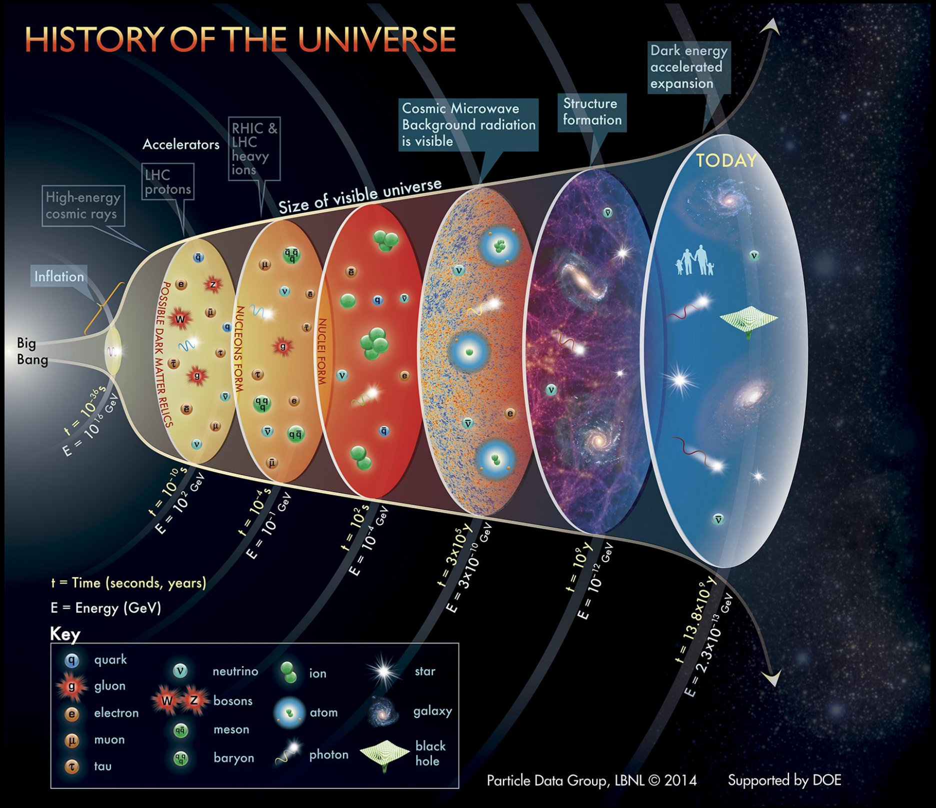



Daniel Baumann, Cosmology, Cambridge University Press, 2022

## Inflation

- An era of exponential expansion of space in the early Universe.
- Introduced to solve Horizon and Flatness problems.
- Also provides a natural explanation to initial density fluctuations.
- These initial density fluctuations arise due to the quantum mechanical nature of the field which causes inflation or some other field present during inflation.
- As different modes cross the horizon, the nature of fluctuations over these modes becomes classical.

# A brief history of the Universe




LBNL Particle Data Group, 2014

Probes for Early Universe

- via photons
  - CMB anisotropies, spectral distortions
- via neutrinos
- via gravitational waves
  - by direct detections of GWs  Lecture by Chiara
  - by constraints on extra degrees of freedom from CMB

# A simple model of inflation

Standard – A single canonical scalar fie
minimally coupled to gravity

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi),\ p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

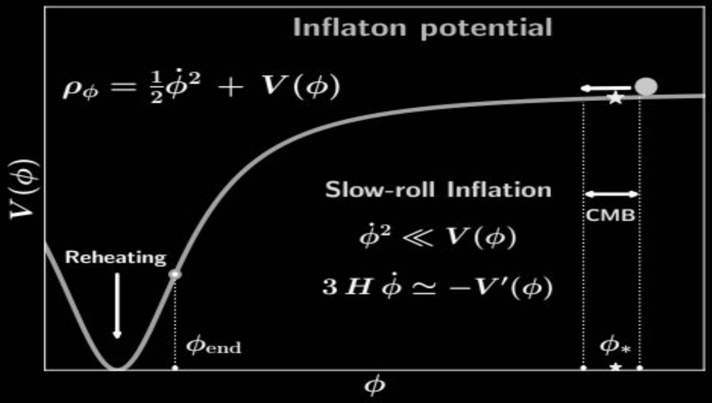
And Einstein's equations imply

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \left(\frac{8\pi G}{3}\right)\rho_\phi,$$

$$\dot{H} = -\frac{1}{2m_p^2}\dot{\phi}^2$$

$$\frac{\ddot{a}}{a} = -\left(\frac{4\pi G}{3}\right)(\rho_\phi + 3\,p_\phi)$$

$$\frac{\ddot{a}}{a} = H^2\,(1-\epsilon_H)$$




Condition for Inflation

$$\epsilon_H = -\frac{\dot{H}}{H^2} < 1 \Rightarrow \dot{\phi}^2 < V(\phi)$$

The dynamics of the scalar field is governed by

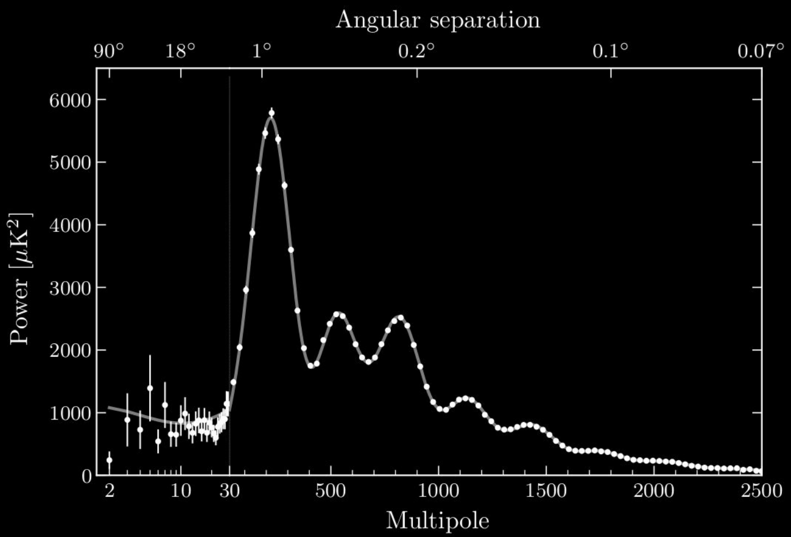
$$\ddot{\phi} + 3\,H\dot{\phi} + V'(\phi) = 0$$

Slide from Swagat S. Mishra's talk

# Testing inflationary models

## The inhomogeneous Universe

- Quantum fluctuations of the inflaton field are stretched to cosmic scales.
- These fluctuations produce curvature (scalar) and gravitational wave (tensor) perturbations.
- Scalar perturbations seed structure formation (galaxies, CMB anisotropies).
- Tensor perturbations produce B-mode polarization in the CMB.




Daniel Baumann, Cosmology, Cambridge University Press, 2022

# Testing inflationary models

## The inhomogeneous Universe

$$ds^2 = a^2(\eta)\left[-(1+2\Psi(\vec{x},\eta)d\eta^2 + ((1-2\Phi)\delta_{ij} + 2h_{ij})dx^i dx^j\right]$$

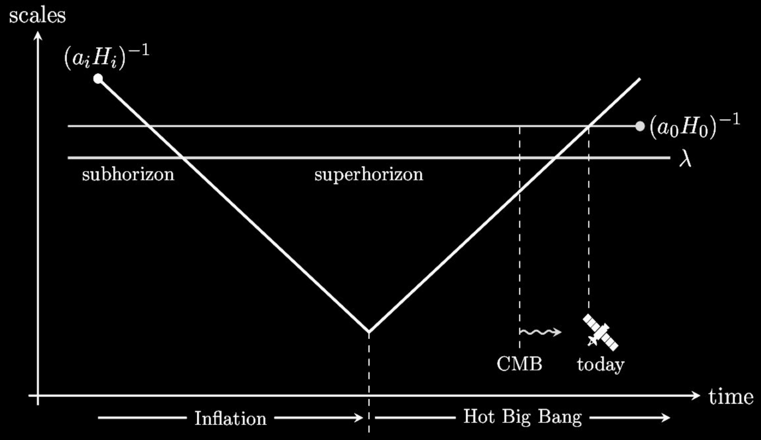
- Evolution of scalar perturbation

$$(a\delta\phi)'' + \left(k^2 - \frac{z''}{z}\right)(a\delta\phi) == 0 \quad \text{where, } z = \frac{a\bar{\phi}'}{\mathcal{H}}$$

- Gauge Invariant scalar perturbation

$$\zeta = \Phi + \frac{H\delta\phi}{\dot{\phi}}$$

- Power spectrum of scalar perturbations:
- $$P_\zeta \propto k^{n_s - 1}$$
- The spectral index n_s characterizes the scale dependence of perturbations.
- The tensor-to-scalar ratio: $r = P_t / P_\zeta$.

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# Testing inflationary models

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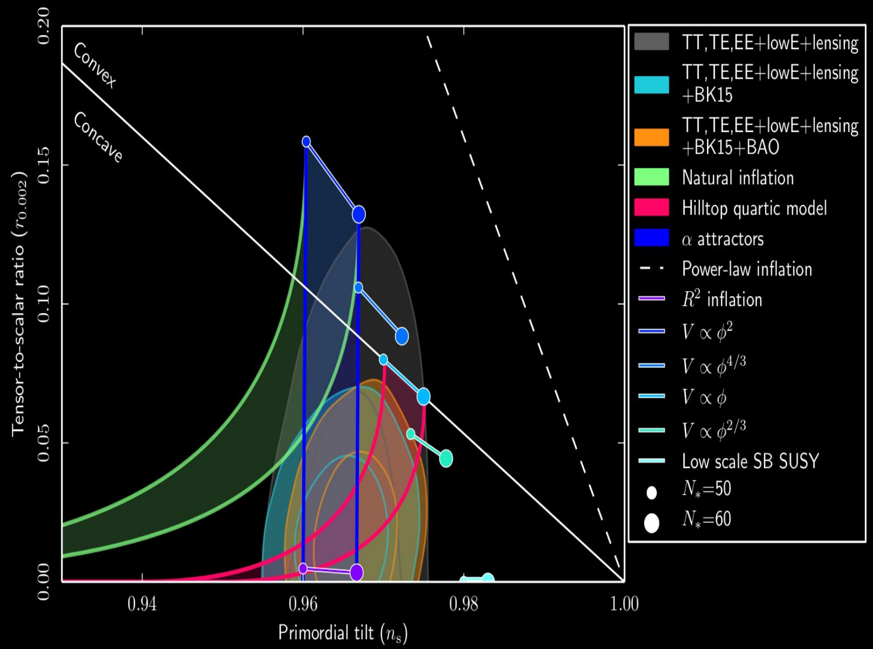


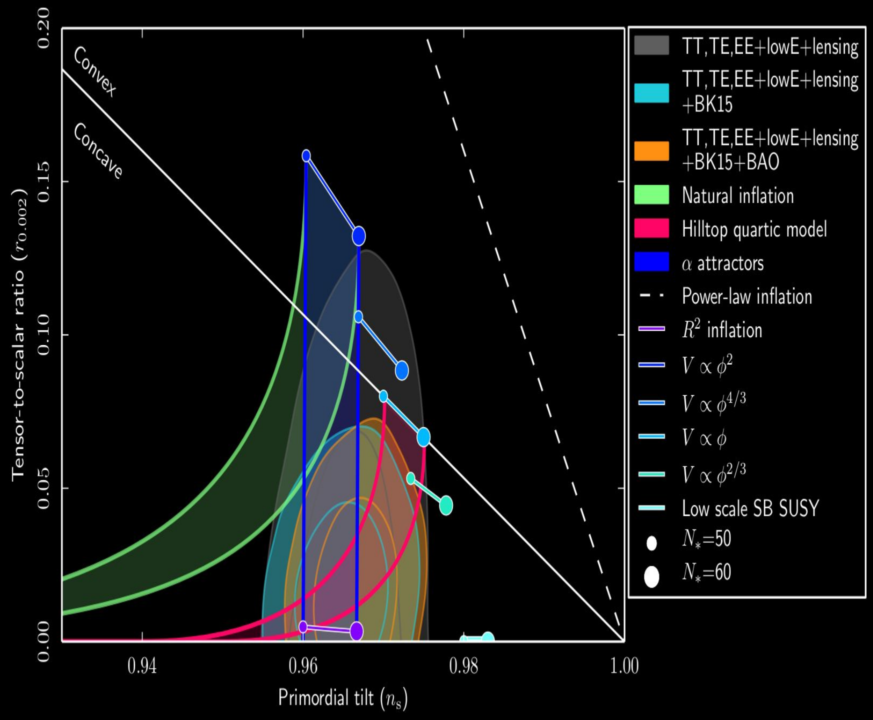

Planck results 2018

# Axion-U(1) Inflation

- Flatness of the potential is protected due to shift symmetry

- First model suggested by the name Natural Inflation
  K. Freese, J. A. Frieman and A. V. Olinto, PRL 1990
  $$V(\phi) = \Lambda^4 \left(1 + \cos\left(\frac{\phi}{f}\right)\right)$$

- Various scenarios has been suggested to make it compatible with the CMB observations




$$S = \int d^4x \sqrt{-g}\left[\frac{m_{\rm Pl}^2}{16\pi} R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\alpha}{4f}\phi F_{\mu\nu}\tilde{F}^{\mu\nu}\right]$$

Planck results 2018

Anber and Sorbo 2009

# Axion-U(1) Inflation : dynamics

- By neglecting the inhomogeneity of axion

$$\left(\partial_\eta^2 + k^2 \mp 2\xi(\mathcal{H}\eta)\frac{k}{\eta}\right) A_k^\pm = 0\,, \qquad \text{where} \qquad \xi = -\frac{\alpha}{2f}\,\frac{\phi'}{\mathcal{H}}$$

- $$A_k^+ \simeq \frac{1}{\sqrt{2\,k}}\left(\frac{k}{2\,\xi\,a\,H}\right)^{1/4} e^{\pi\xi - 2\sqrt{2\xi k/(aH)}}\,, \qquad \text{For, } (8\xi)^{-1} \leq k/(aH) \leq 2\xi$$

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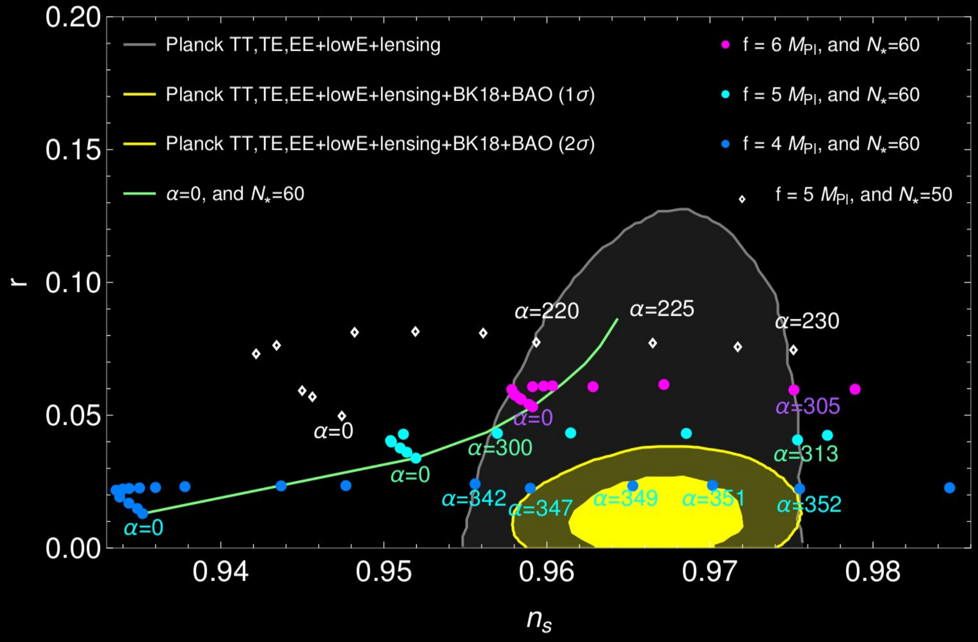
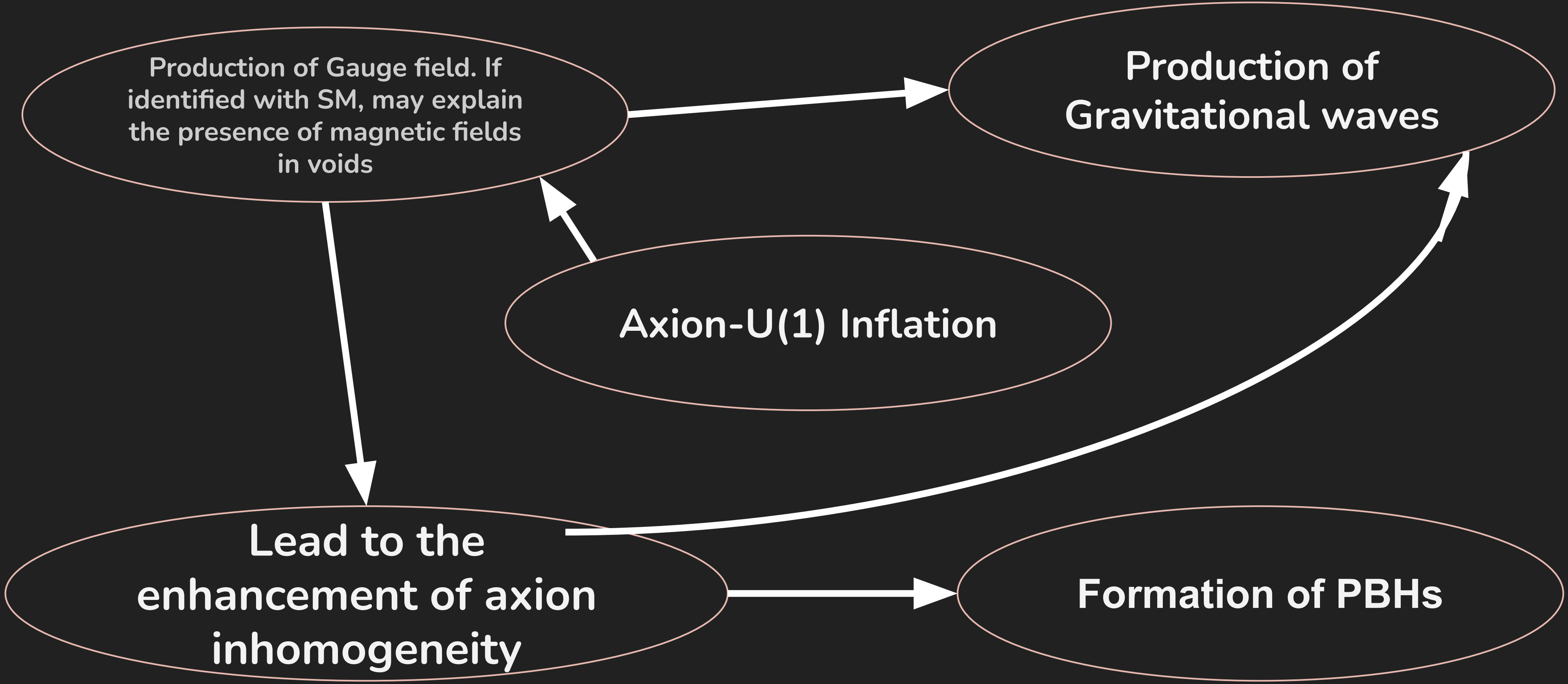
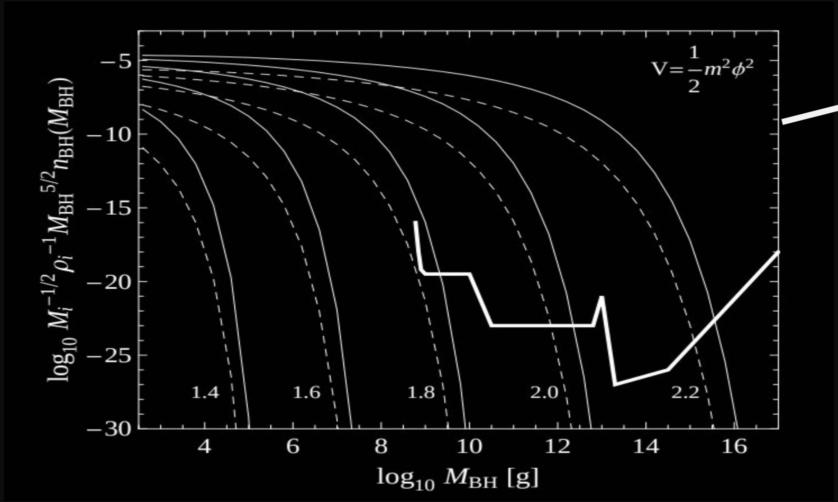



K. Alam, K. Dutta and N. Jamana 2024

# Axion-U(1) Inflation : Phenomenology



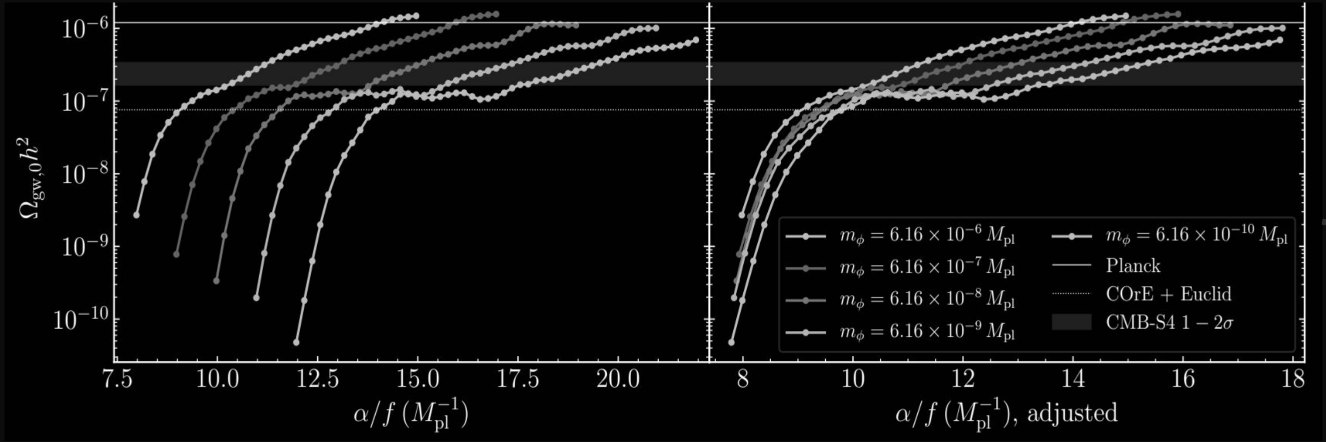
# Constraints on the coupling between axion and gauge field




Assume chi square PDF

$\frac{\alpha}{f} < 130/m_{pl}$

E. Bugaev and P. Klimai, PRD 2014




Adshead et al, PRD 2020
Adshead el al, PRL 2020

$\frac{\alpha}{f} < 70/m_{pl}$

for $m_\phi = 6.16 \times 10^{-6} M_{pl}$

# Lattice simulations of Axion-U(1) Inflation

- We use pencil code to solve the axion-U(1) setup. Equations are begin solved

$$\phi'' + 2\mathcal{H}\phi' - \nabla^2\phi + a^2\frac{dV}{d\phi} = \frac{\alpha}{f}\frac{1}{a^2}\mathbf{E}\cdot\mathbf{B}\,,$$

$$\mathbf{A}'' - \boldsymbol{\nabla} A_0' - \nabla^2\mathbf{A} + \boldsymbol{\nabla}(\boldsymbol{\nabla}\cdot\mathbf{A}) - \frac{\alpha}{f}\left(\phi'\mathbf{B} + \boldsymbol{\nabla}\phi\times\mathbf{E}\right) = 0\,,$$

→ backreact_infl.f90

→ disp_current.f90

Along with the FLRW background.

$$\mathcal{H}^2 = \frac{8\pi}{3m_{\rm pl}^2}a^2\rho$$

$$\left\langle \frac{1}{2}\frac{\phi'^2}{a^2} + \frac{1}{2}\frac{(\nabla\phi)^2}{a^2} + V(\phi) + \frac{E^2+B^2}{2a^4}\right\rangle$$

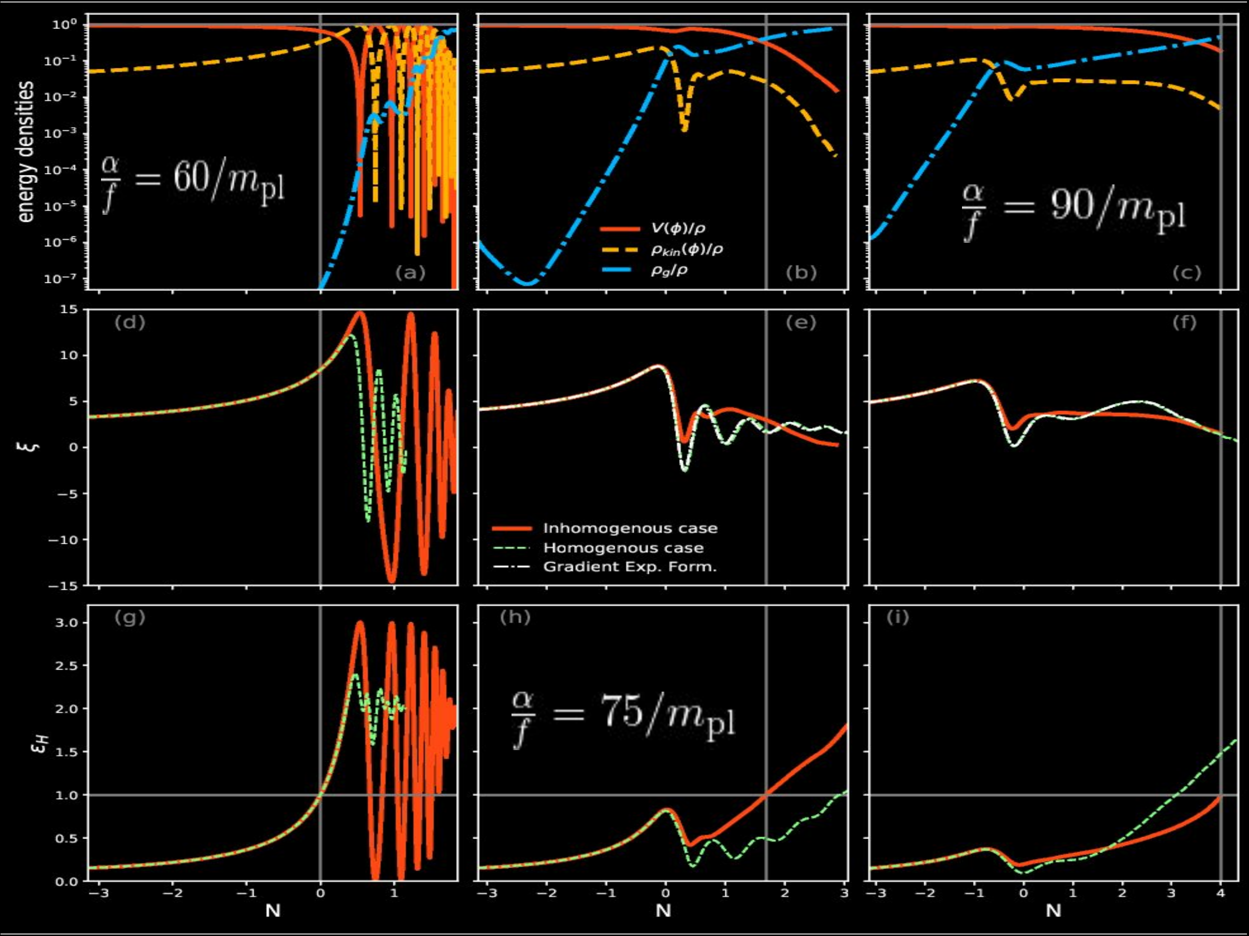
→ backreact_infl.f90

# Check for constraint equation

- We have two gauge choices to be used
  - Lorentz Gauge  (set llongitudinalE=.false. And llorenz_gauge_disp=.true. )
  - Weyl Gauge, $A_0$=0 (by default this one is selected)

$$C.E. = \left\langle \frac{\nabla \cdot E - \frac{\alpha}{f}\nabla\phi \cdot B}{\sqrt{(\nabla \cdot E)^2 + (\frac{\alpha}{f}\nabla\phi \cdot B)^2}} \right\rangle.$$

# Axion-U(1) Inflation : dynamics from lattice simulations

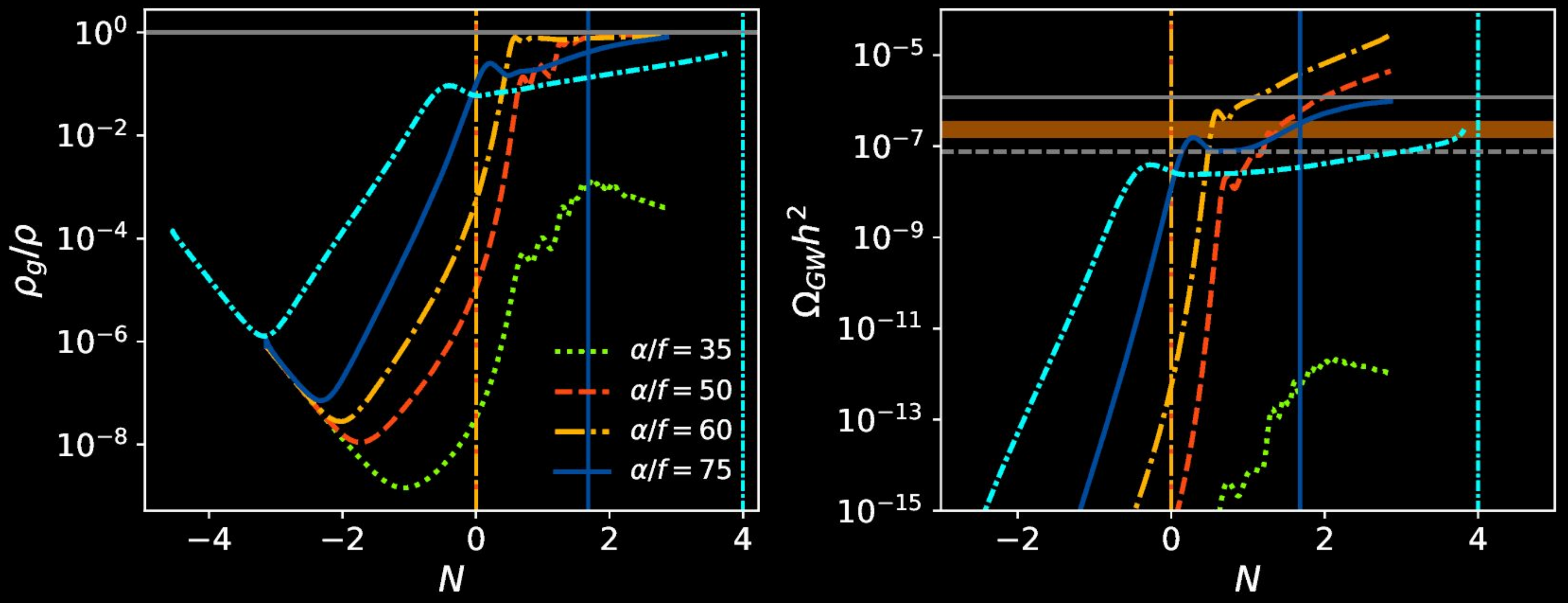



RS, AB, KS, AV, Arxiv 2411.04854

AB - Axel Brandenburg
KS – Kandaswamy Subramanian
AV - Alex Vikman

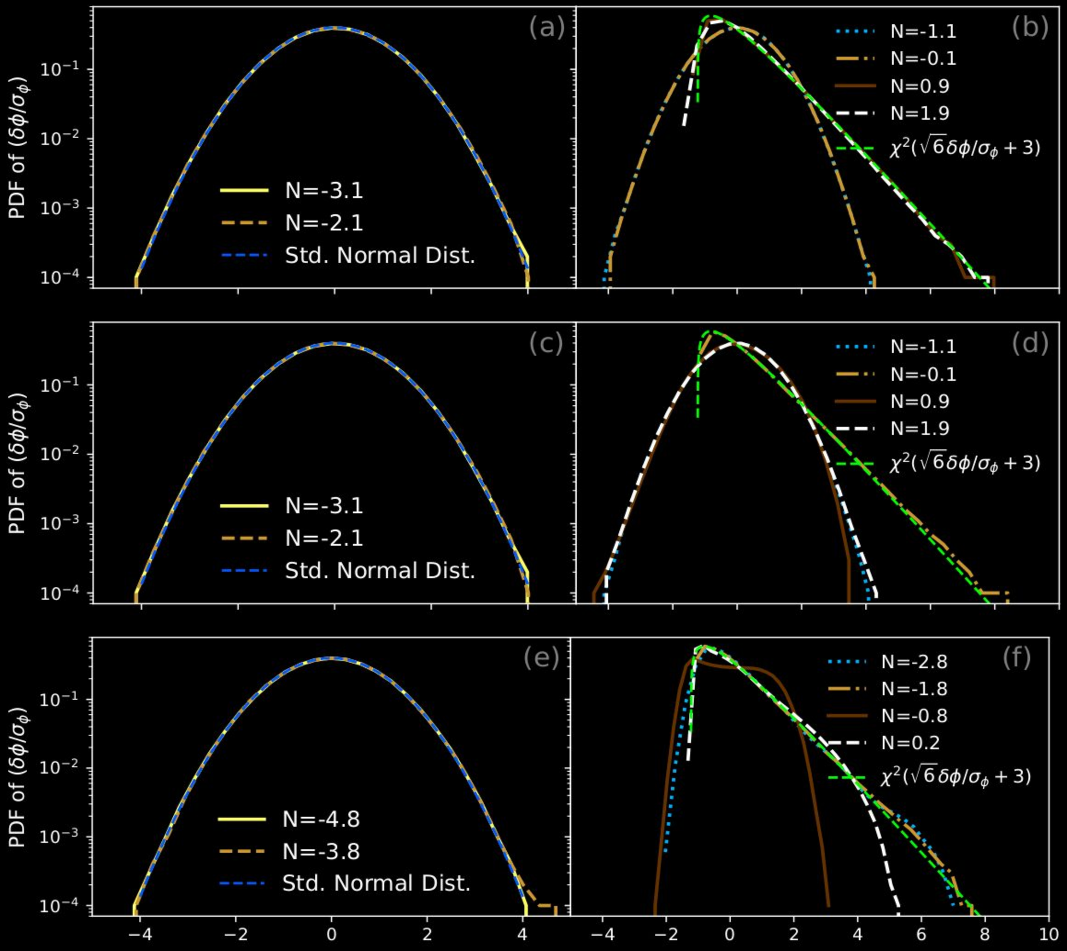
D. G. Figueroa, J. Lizarraga, A. Urio and J. Urrestilla, PRL 2023

# Energy budget of gauge field and produced GWs




RS, AB, KS, AV, Arxiv 2411.04854

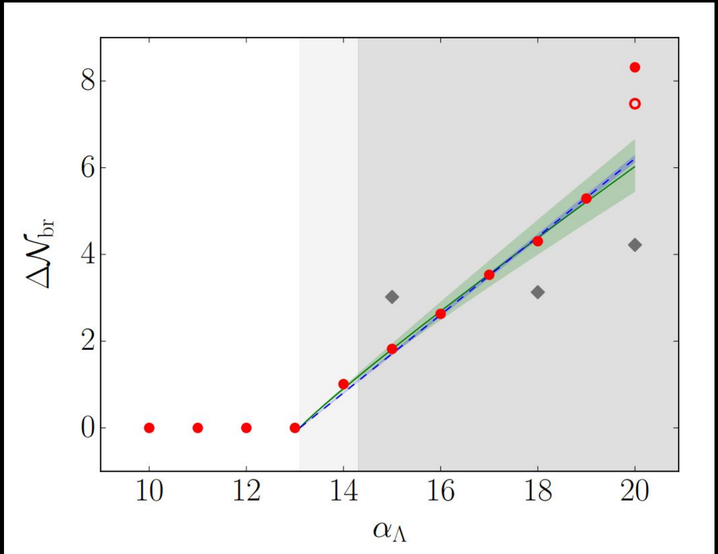
# PDF of the axion fluctuations




RS, AB, KS, AV, Arxiv 2411.04854

A. Caravano, E. Komatsu, K. D. Lozanov and J. Weller, PRD 2022

# Extended duration of inflation due to backreaction




| $\alpha_\Lambda$ | $\Delta\mathcal{N}_{br}$ | | | |
|---|---|---|---|---|
| | linear (77) | power-law (78) | linear (79) | power-law (80) |
| 20 | $6.21\pm0.07$ | $6.03^{+0.61}_{-0.57}$ | $5.9\pm0.1$ | $5.85^{+0.45}_{-0.45}$ |
| 22.5 | $8.46\pm0.09$ | $8.04^{+0.90}_{-0.83}$ | $7.9\pm0.2$ | $8.88^{+0.81}_{-0.77}$ |
| 25 | $10.7\pm0.1$ | $10.0^{+1.21}_{-1.08}$ | $9.9\pm0.2$ | $12.06^{+1.20}_{-1.12}$ |
| 30 | $15.2\pm0.2$ | $13.9^{+1.80}_{-1.67}$ | $13.8\pm0.3$ | $18.75^{+2.11}_{-1.94}$ |
| 35 | $19.7\pm0.2$ | $17.6^{+2.53}_{-2.16}$ | $17.8\pm0.4$ | $25.76^{+3.15}_{-2.84}$ |

D. G. Figueroa, J. Lizarraga, Nicolas Loayza,
A. Urio and J. Urrestilla, Arxiv: 2411.16368

# Other Implementations

- axion-SU(2) dynamics during inflation OI, EIS, RS and AB, JCAP 2024 &AB, OI, EIS, RS, JCAP 2024
  - $$S=\int d^4x\sqrt{-\det g_{\mu\nu}}\left[\frac{M_{\rm pl}^2}{2}R-\frac{1}{2}(\partial\phi)^2-V(\phi)-\frac{1}{2}(\partial\chi)^2-U(\chi)-\frac{1}{4}F^a_{\mu\nu}F^{a\,\mu\nu}+\frac{\lambda\chi}{4f}F^a_{\mu\nu}\tilde{F}^{a\,\mu\nu}\right],$$
  - 
  - Not the full lattice simulation but solve the background and first order equation in Fourier space
  - Explore the parameter space where the backreaction of the first order part on the background evolution is important

- Schwinger effect in axion inflation on a lattice OI, EIS, AB Arxiv:2506.20538
  - In this study, the authors have including charge currents derived for homogeneous gauge fields

OI- Oksana Iarygina
EIS - Evangelos I. Sfakianakis

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**Thank you** (ramkishor.sharma@uohyd.ac.in)