Pencil Code school on early Universe physics and gravitational waves

October 2025, CERN (Switzerland)

Lecture: First-Order Phase Transitions

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Introduction: phase transitions in the early Universe

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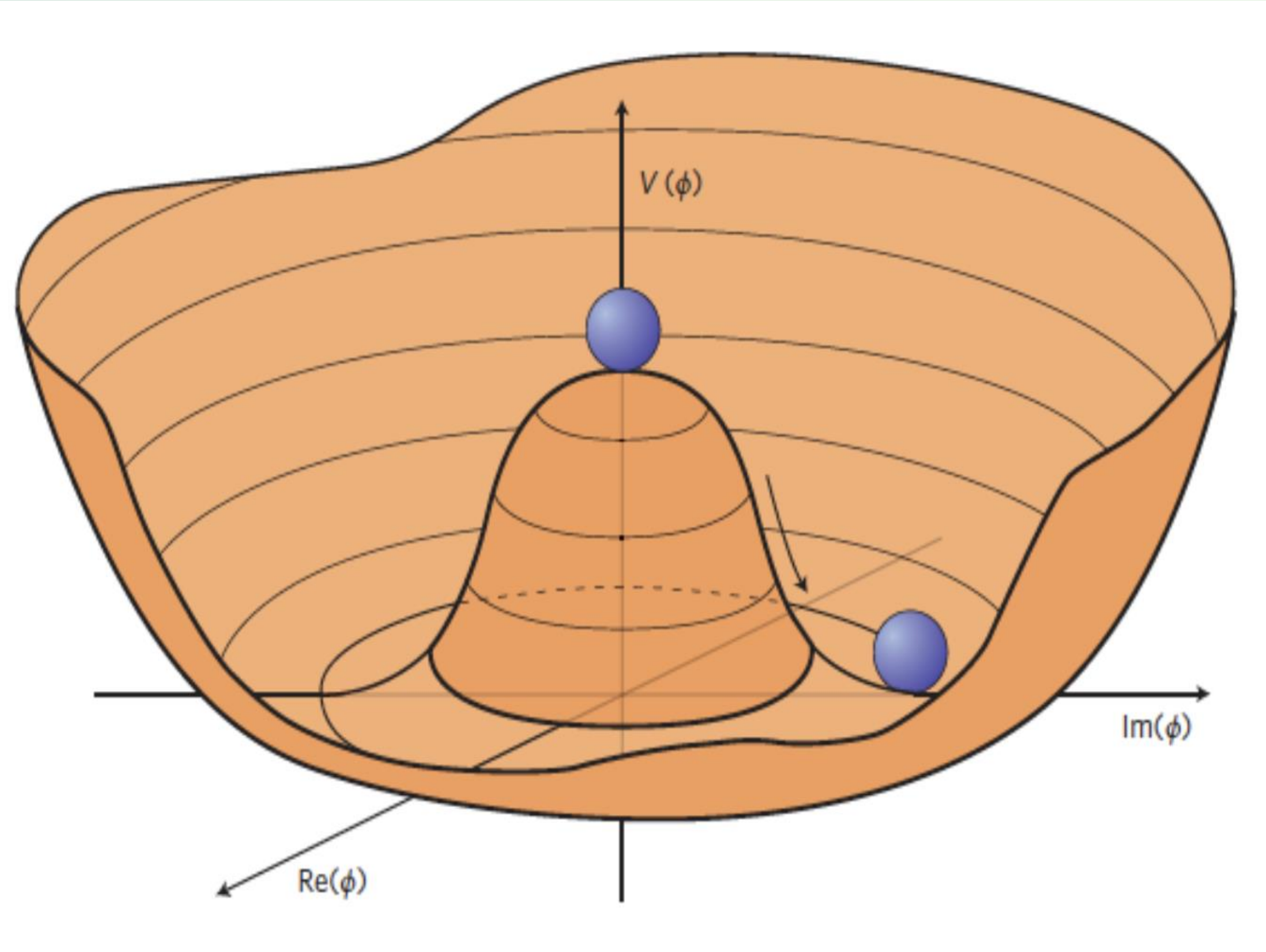

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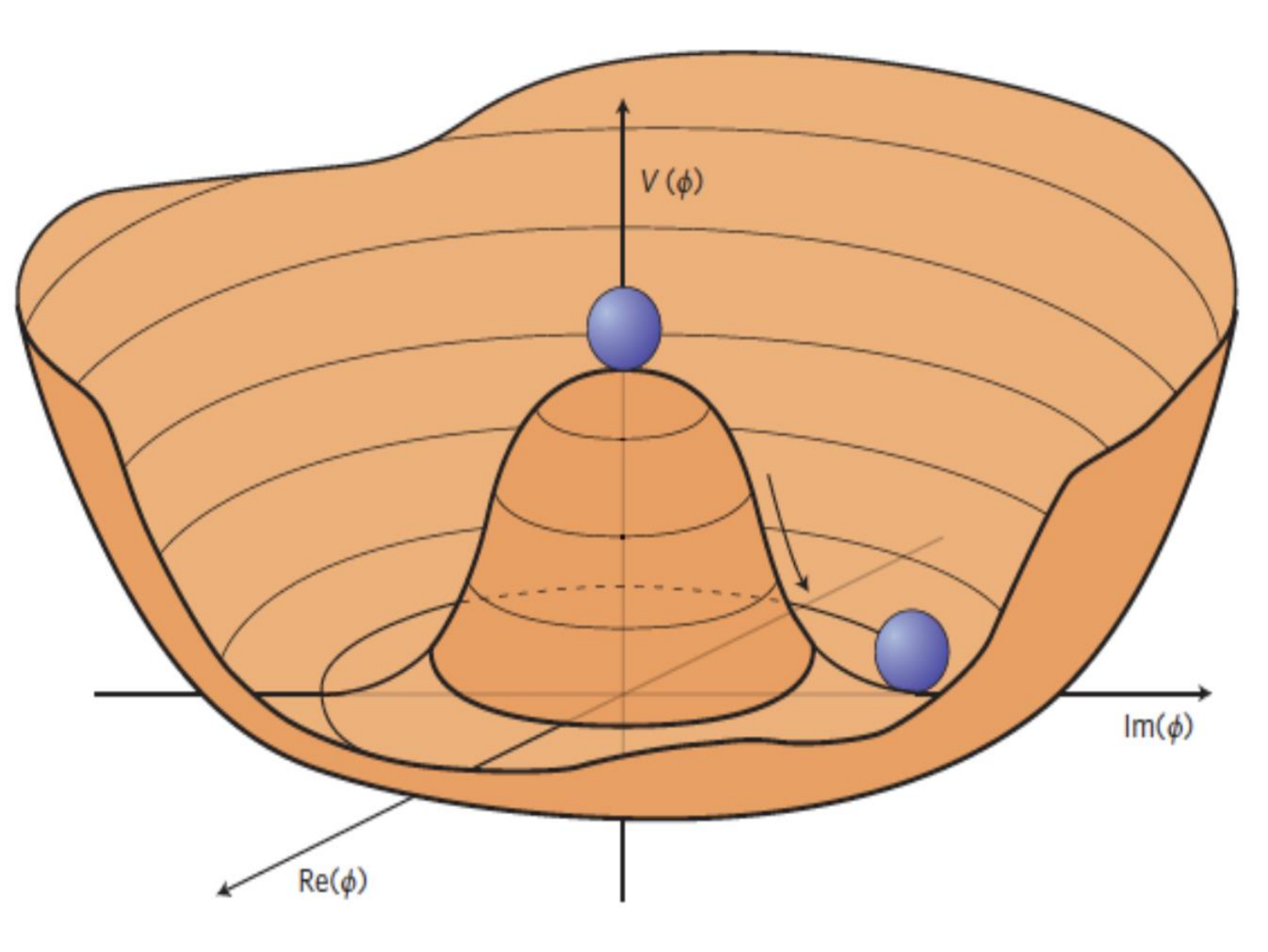

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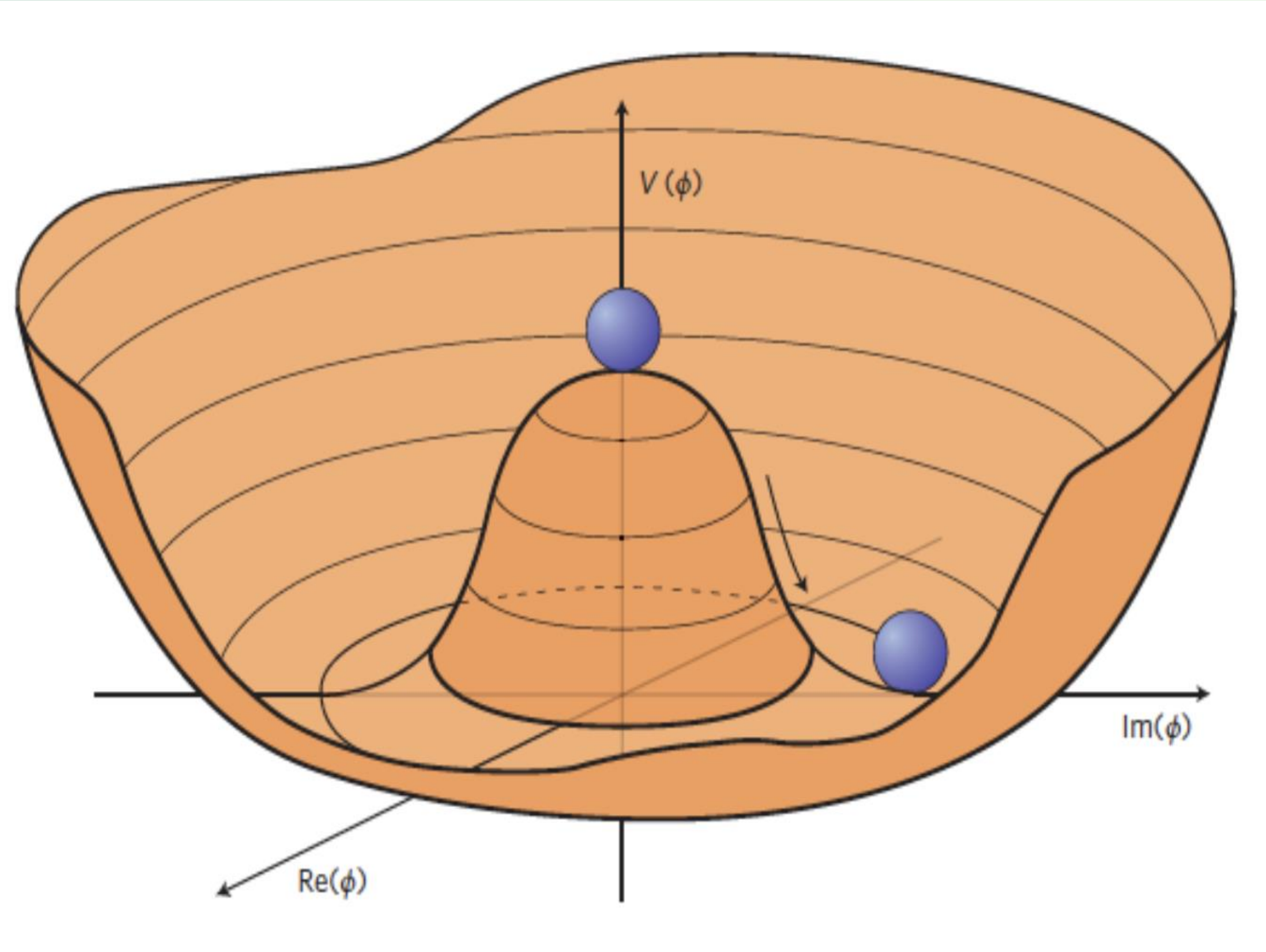
$\xrightarrow{\text{vev}}$ $|\phi| = \sqrt{\mu^2/\lambda^2} \equiv v \neq 0$



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As temperature decreases the Higgs vev goes from zero to $v \neq 0$ → Electroweak Spontaneous Symmetry Breaking (EWSSB)

$$SU(3)_C \bigotimes SU(2)_L \bigotimes U(1)_Y \to SU(3)_C \bigotimes U(1)_{em}$$

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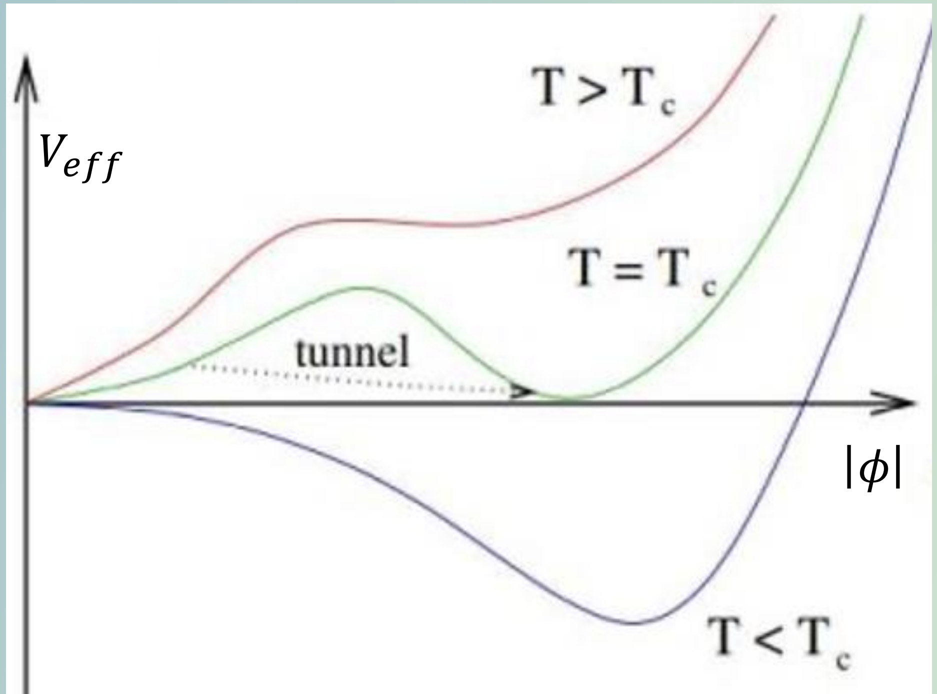

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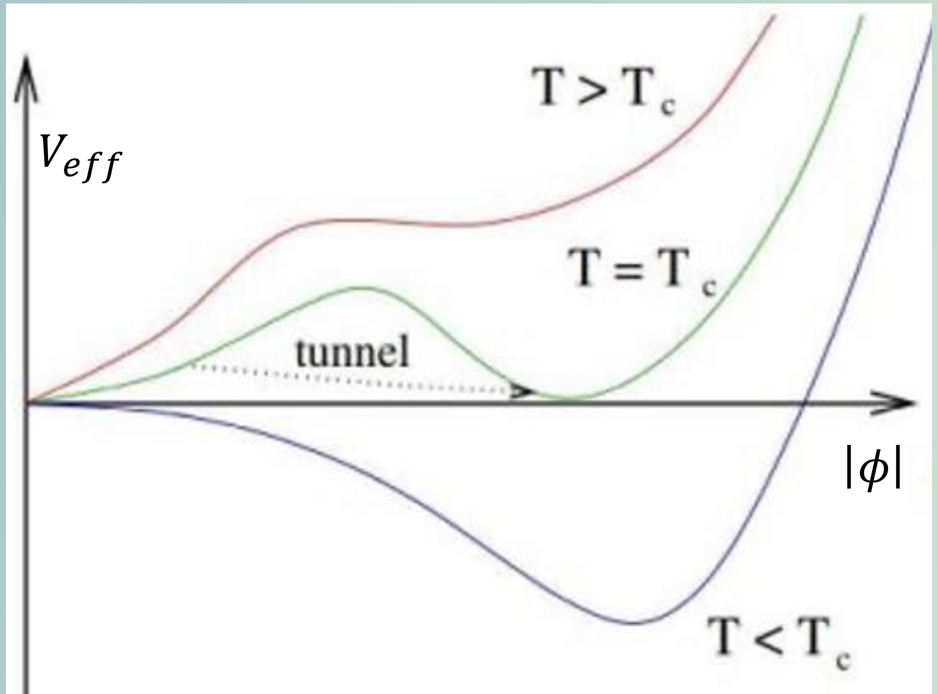

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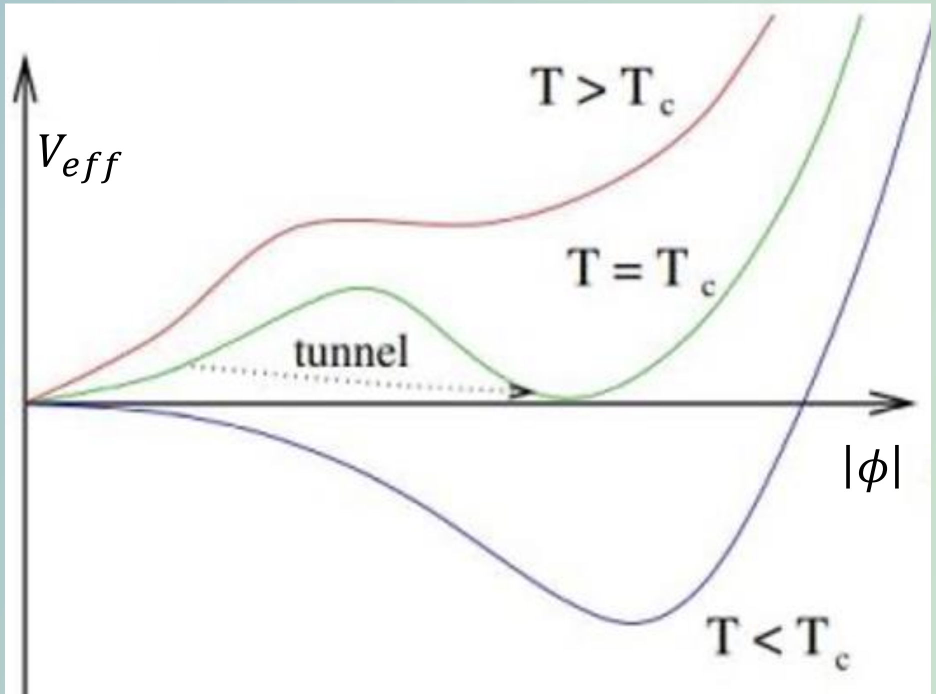

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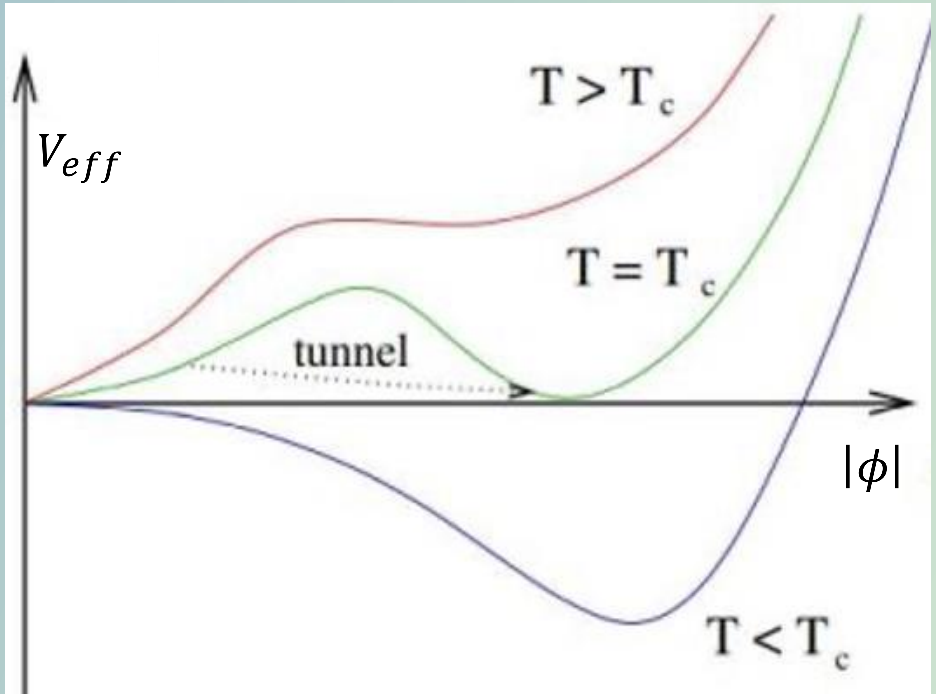

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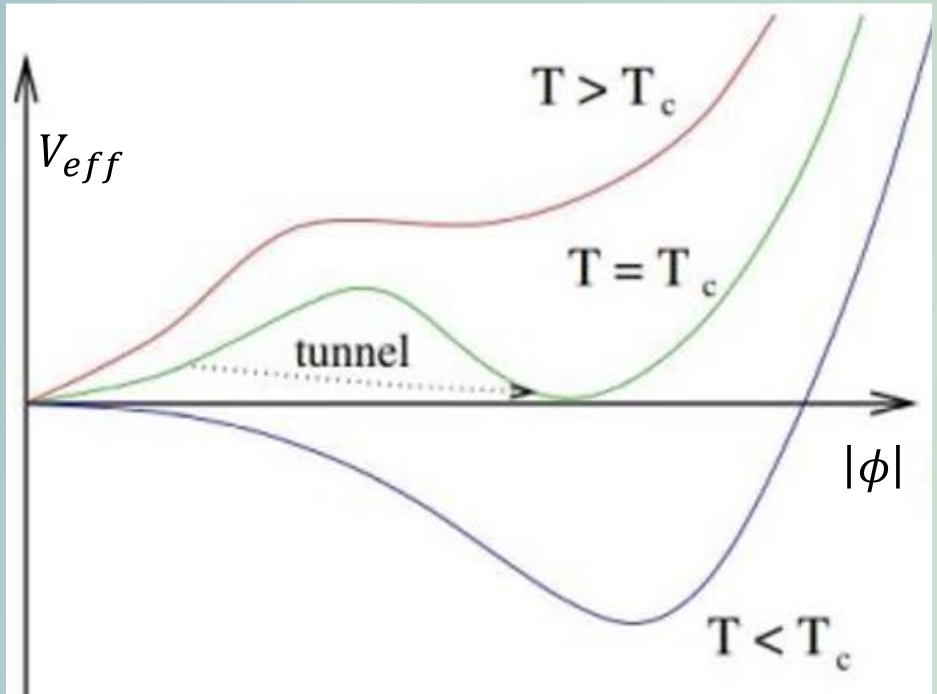
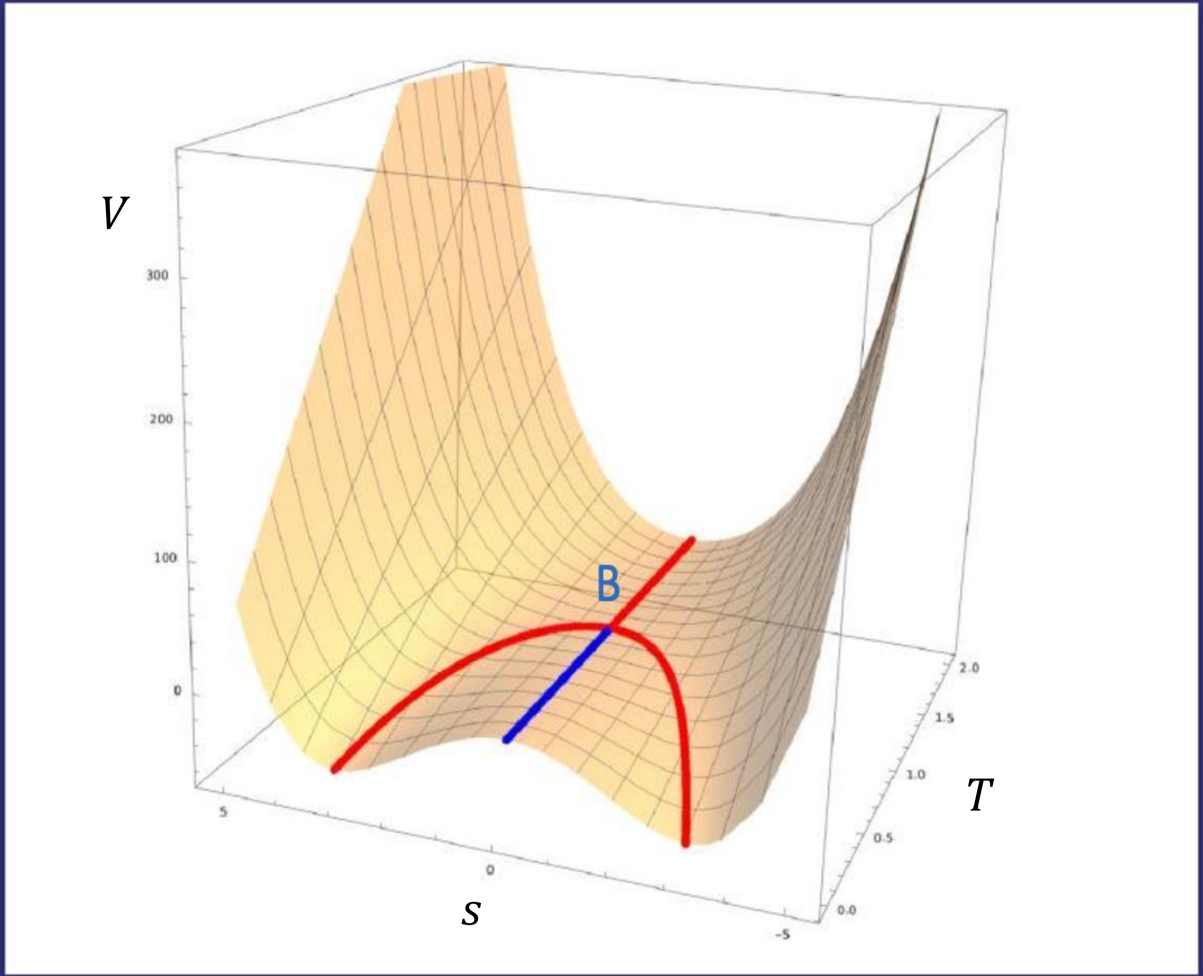


In the Standard Model the EW phase transition is a crossover ($E \neq 0$ but small)

However in BSM theories we can easily have first-order phase transitions (e. g. in SUSY already at tree level)

Second-Order Phase Transition

$$V(T,s) = 10(T-1)s^2 + \frac{s^4}{2}$$



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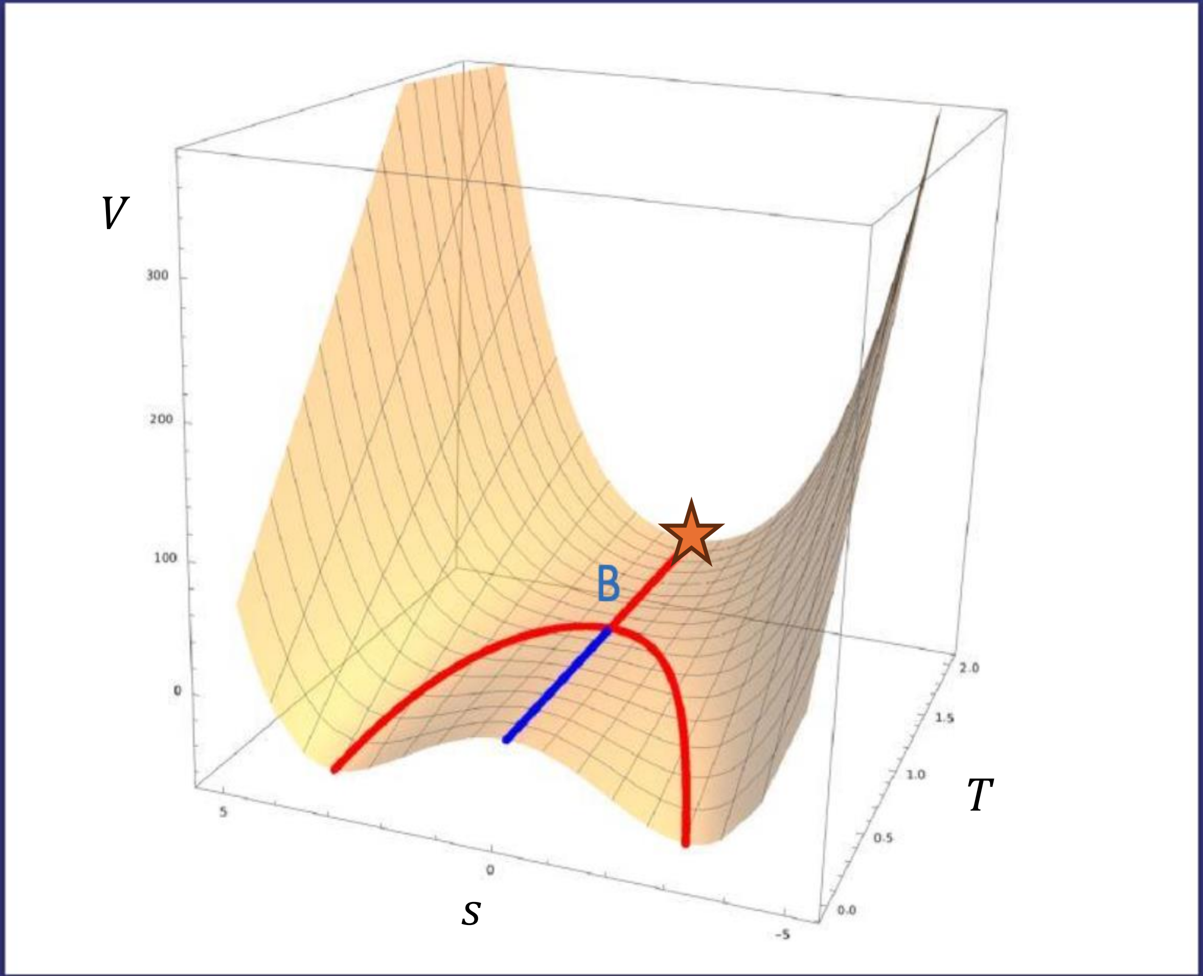
phase transition temperature

$T > T_0$



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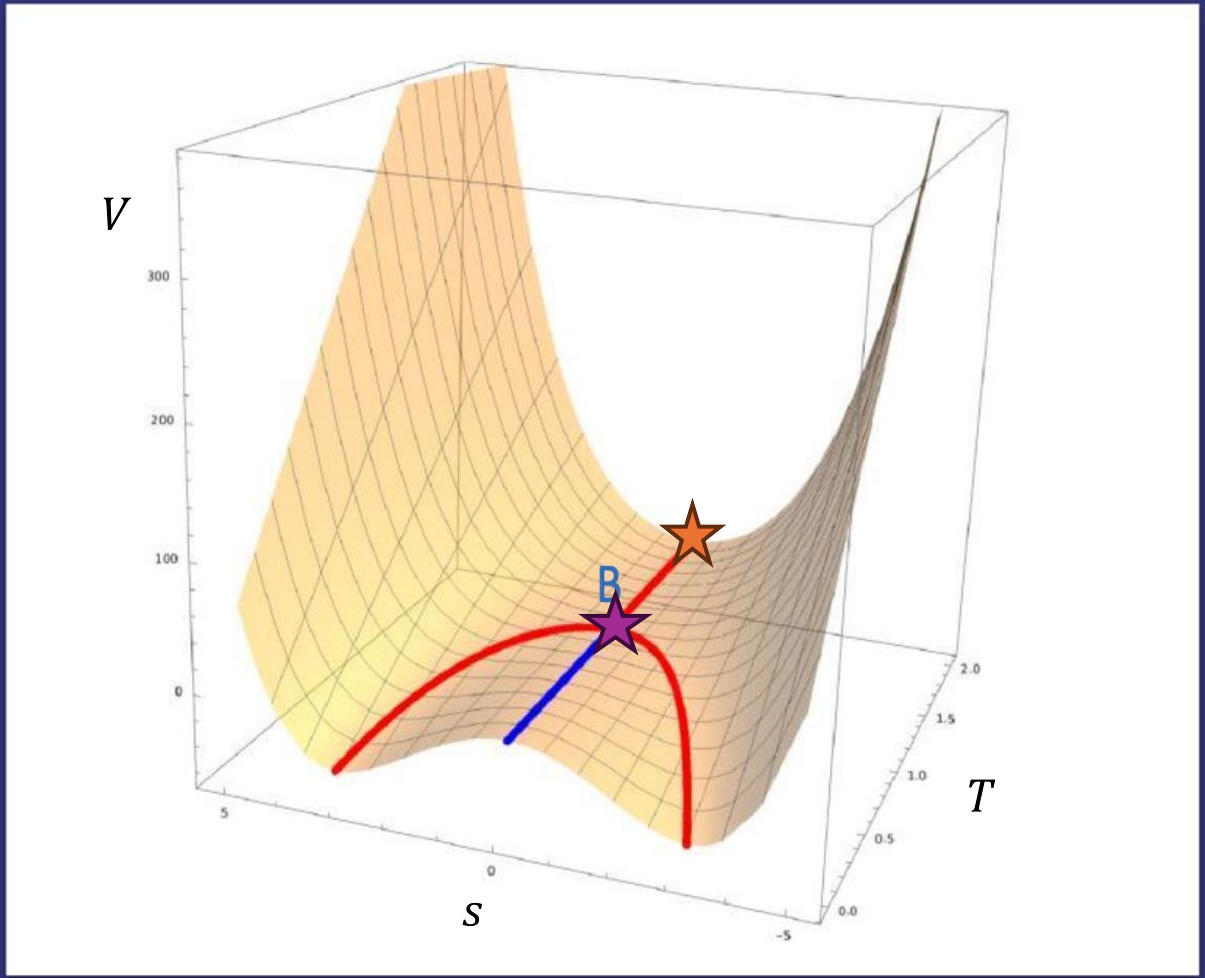

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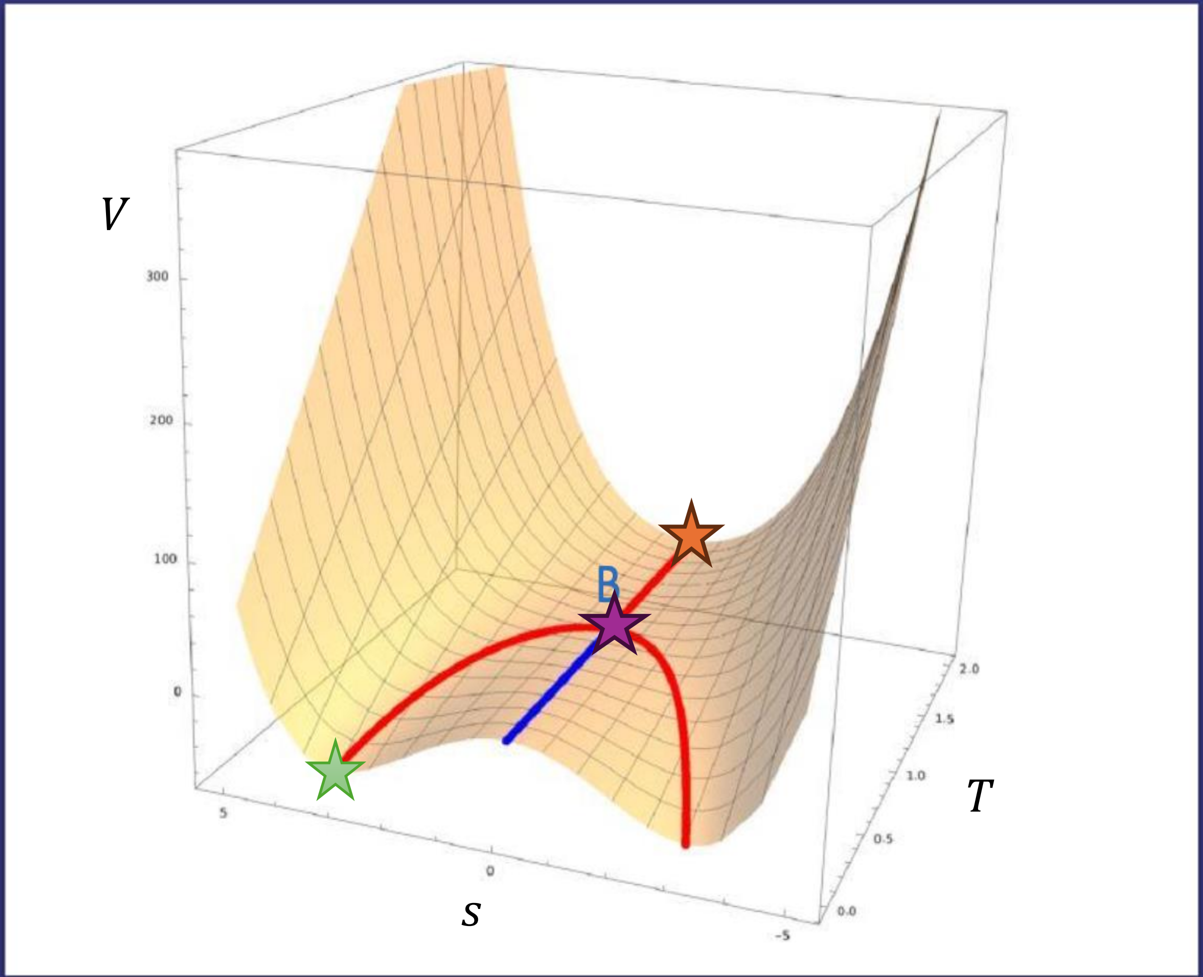

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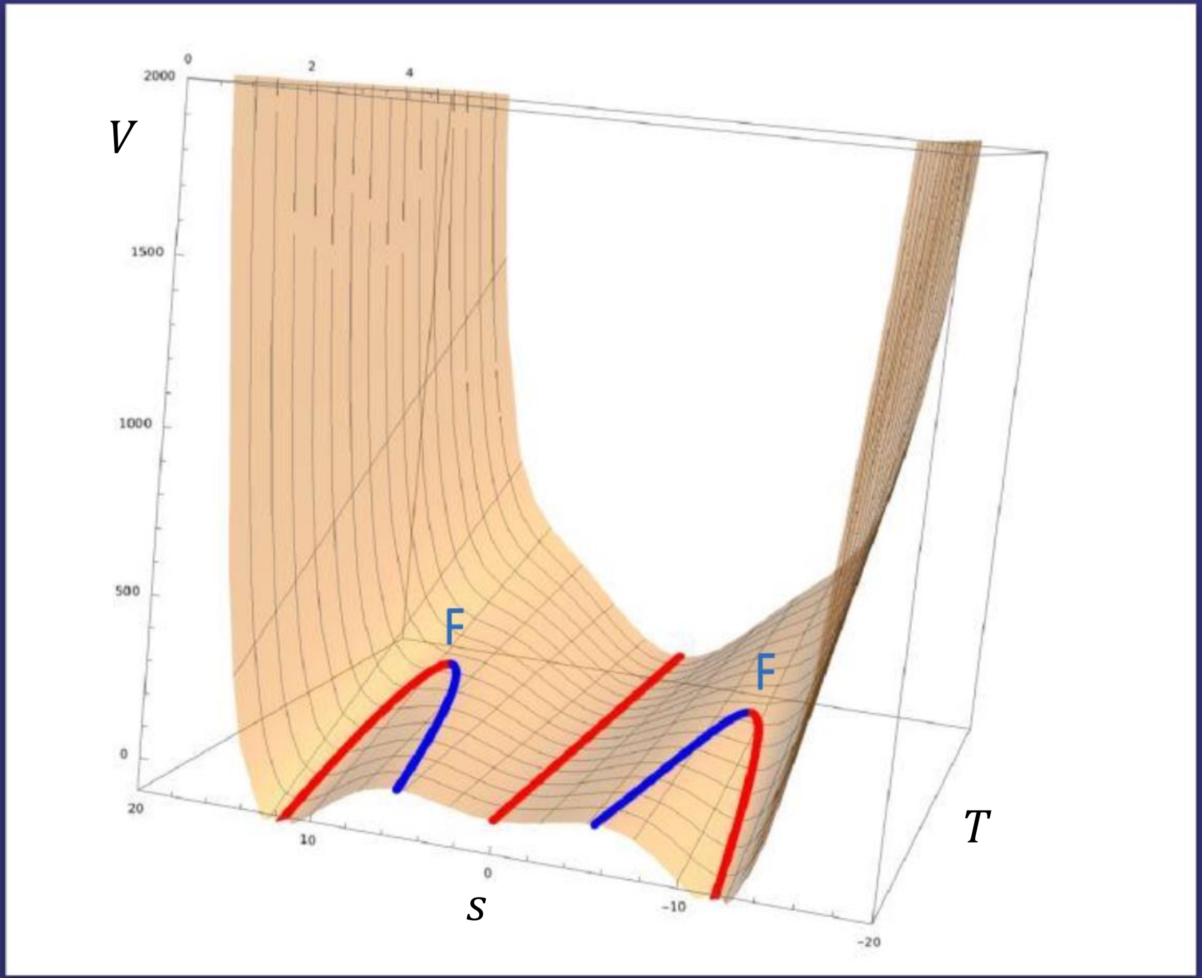

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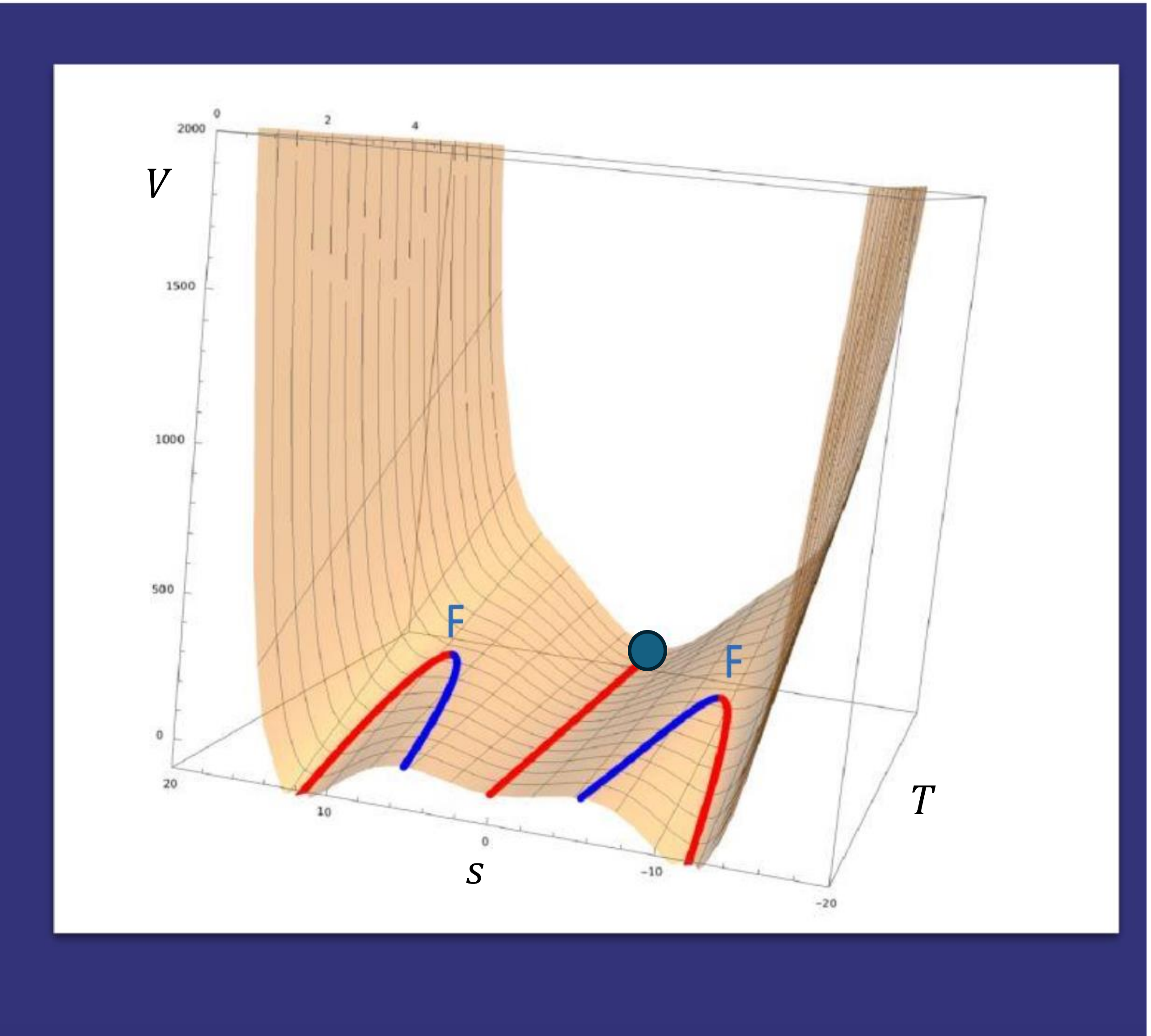

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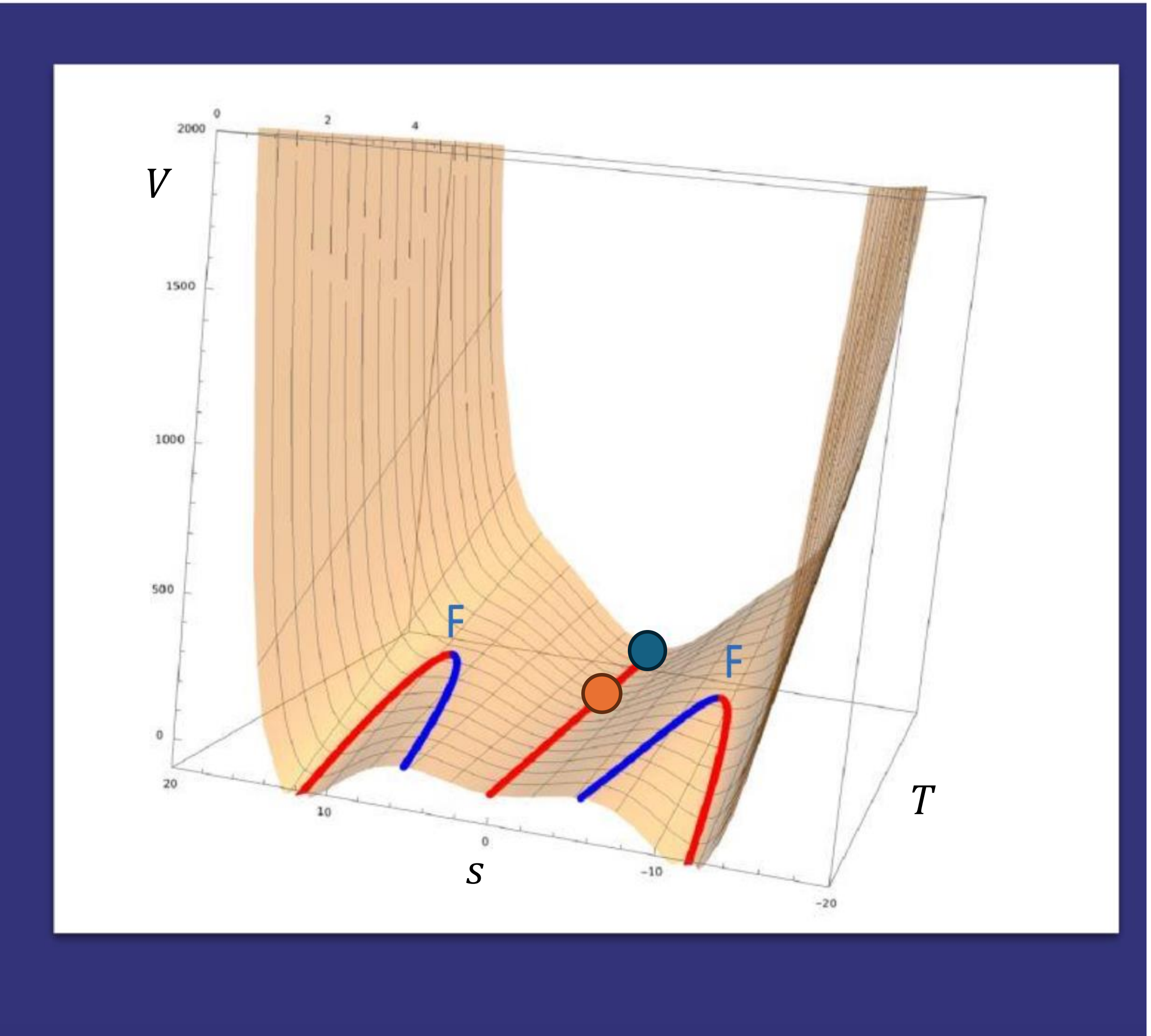

High T

$T = T^*$

Temperature at which local minima appear outside the origin

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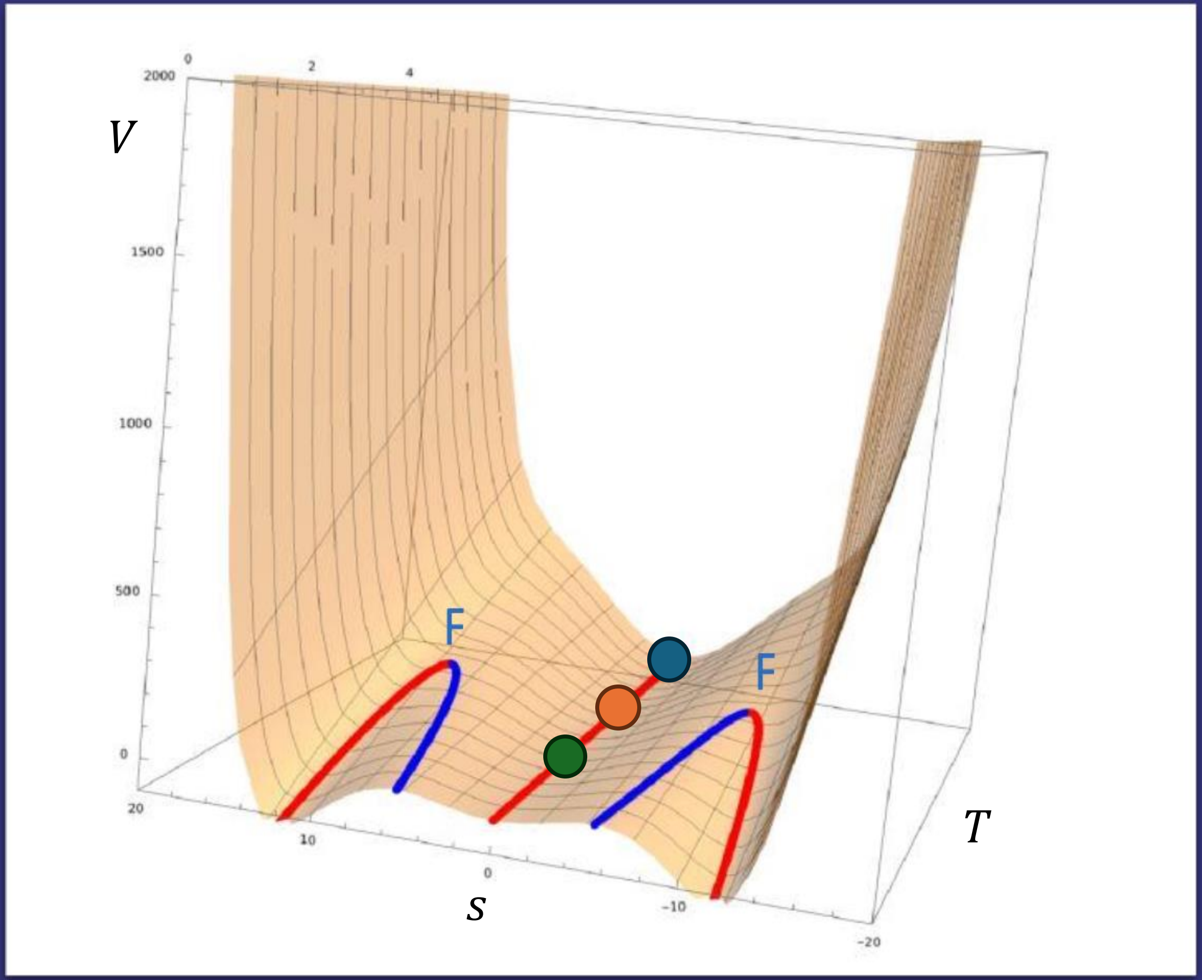

High T

$T = T^*$ — Temperature at which local minima appear outside the origin

$T = T_C$ — Critical temperature (minima outside and at the origin are degenerate)

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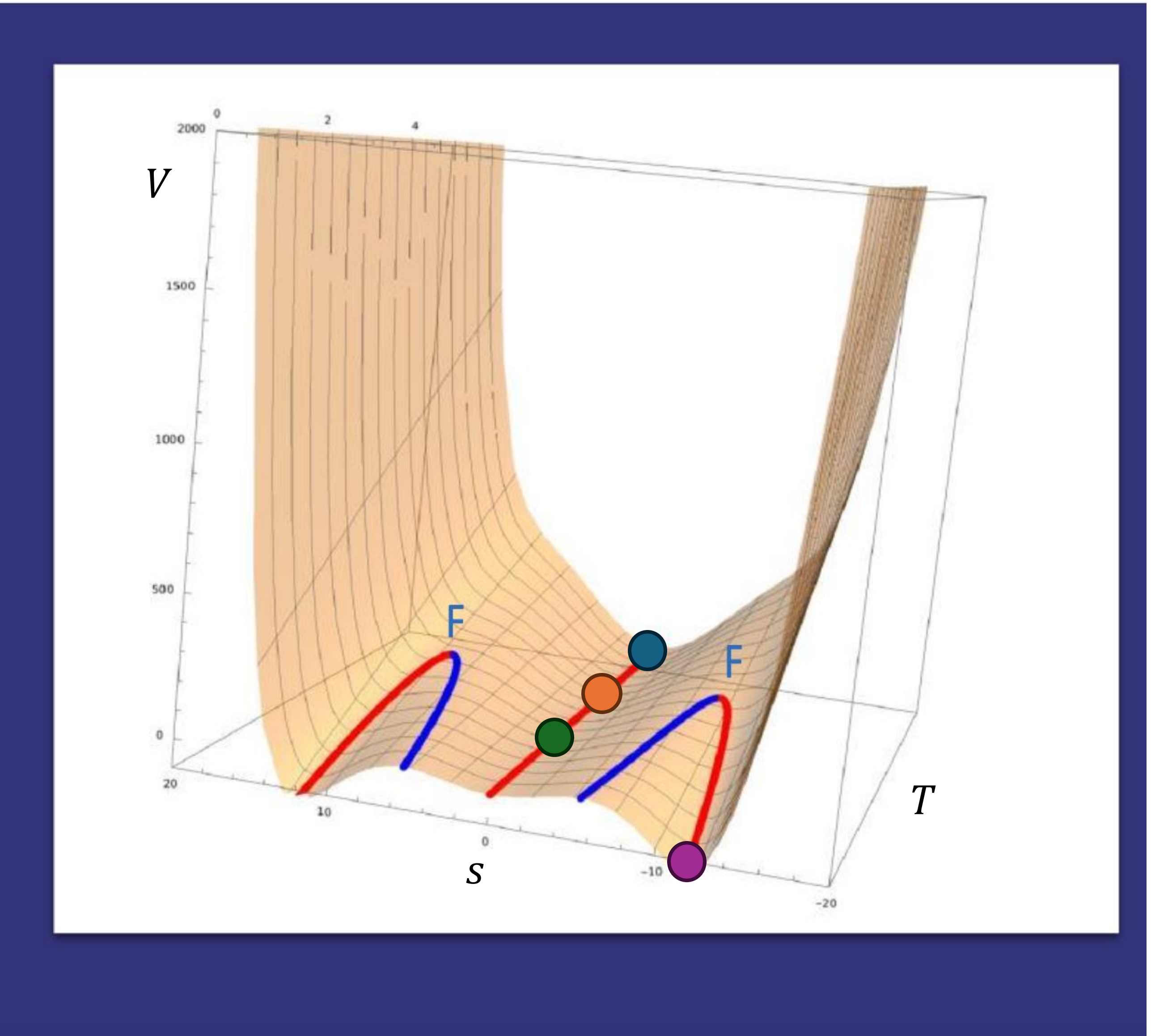

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$T = T_N$ Nucleation temperature (at which the phase transition occurs)

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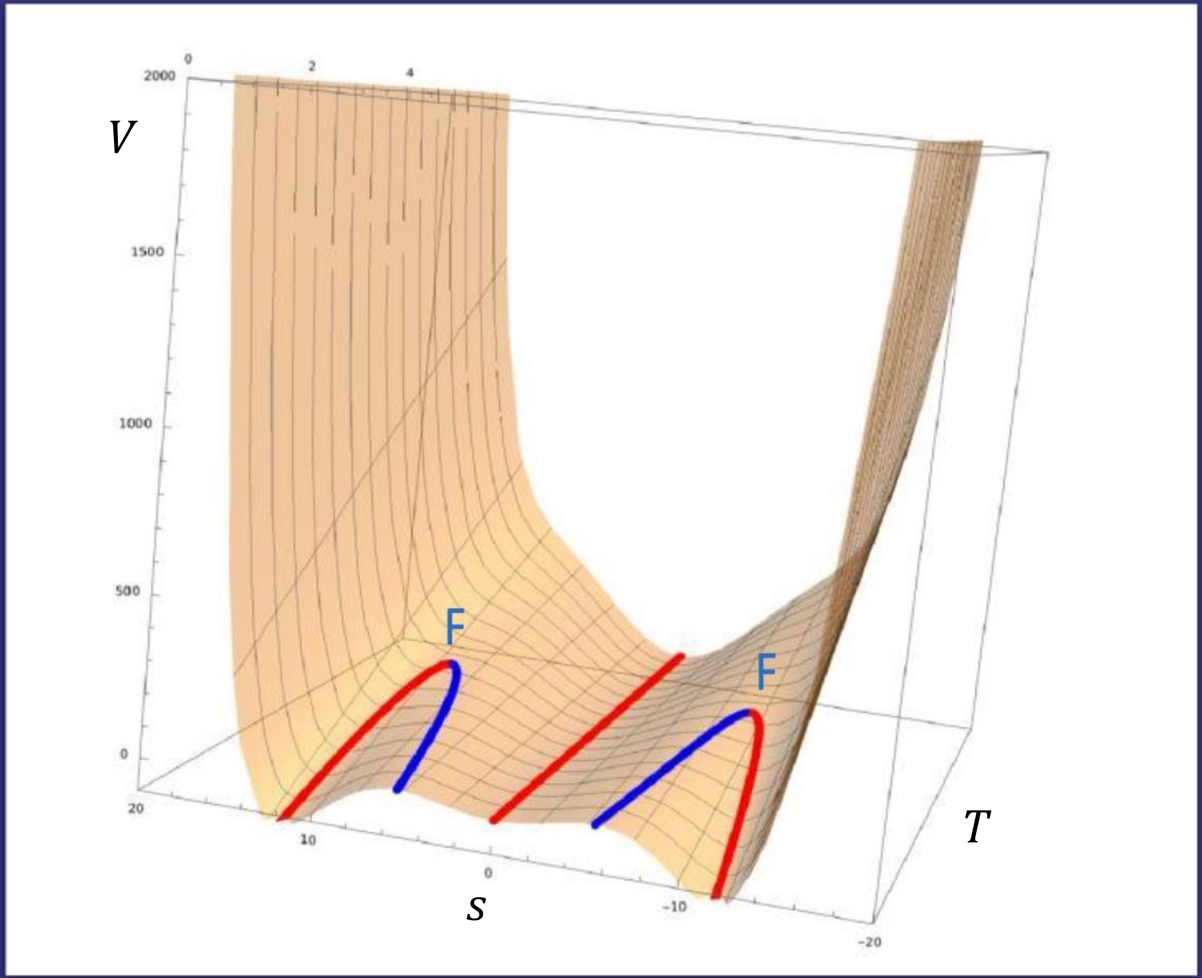

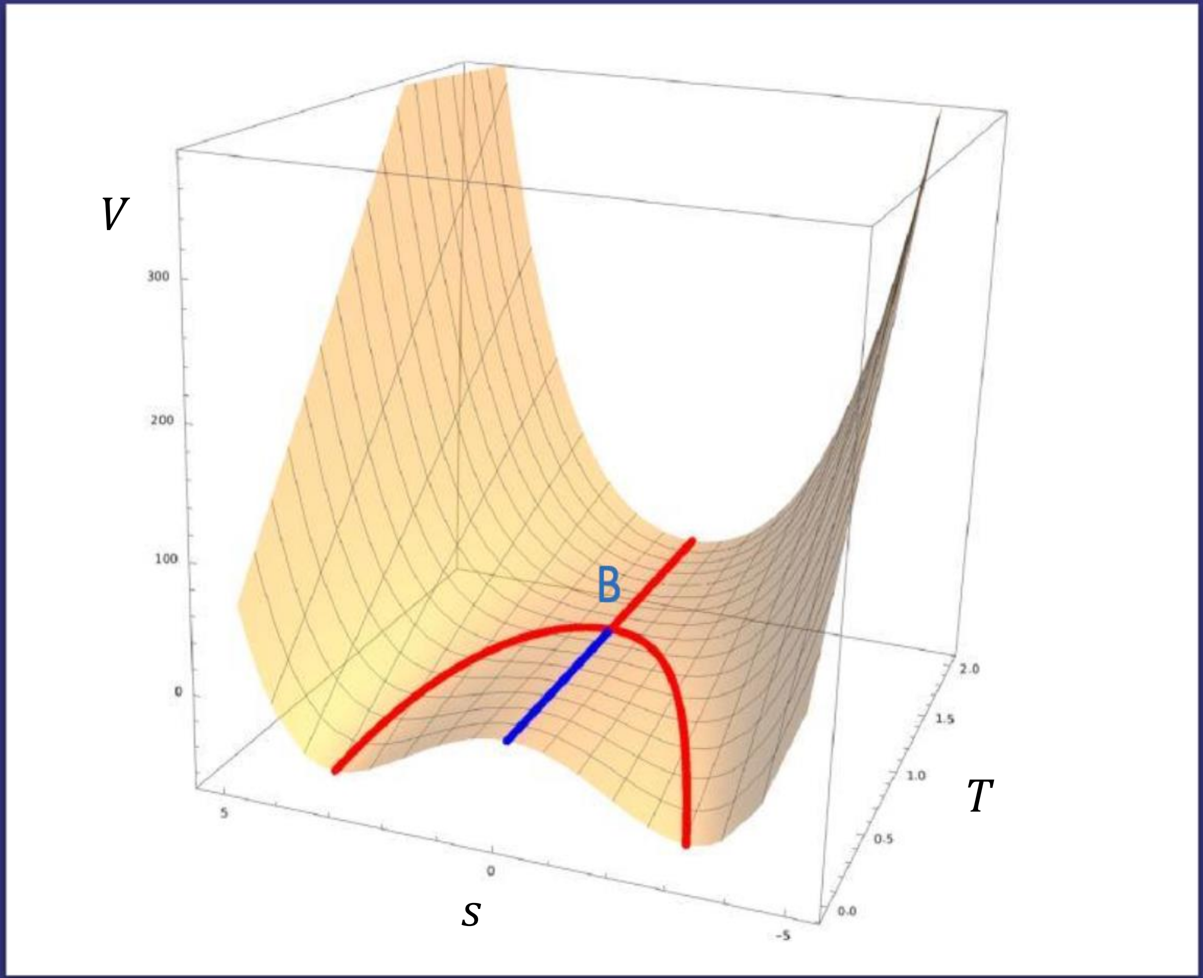

Introduction: first-order phase transitions and baryogenesis

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Explaining matter excess over antimatter requires baryon asymmetry (BAU problem)

$$\frac{n_b - \bar{n}_b}{s} = \frac{1}{7.04}\frac{n_b - \bar{n}_b}{n_\gamma} = \begin{cases} 8.2 - 9.4 \times 10^{-11}, & \text{(BBN)}, \\ 8.65 \pm 0.09 \times 10^{-11}, & \text{(CMB)}. \end{cases}$$

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A. Sakharov (1967) → dynamical baryogenesis mechanism requires three conditions:

1. Baryon number violation.
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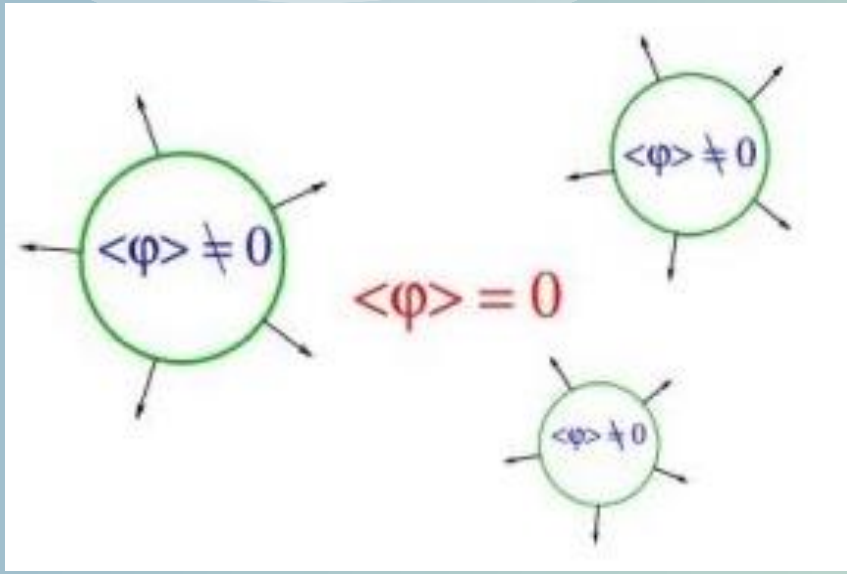
A possible solution → EW baryogenesis

See recent review by Jorinde van de Vis et al. [2508.09989]

Introduction: first-order phase transitions and baryogenesis

First-Order Phase Transitions occur through the nucleation of broken phase bubbles



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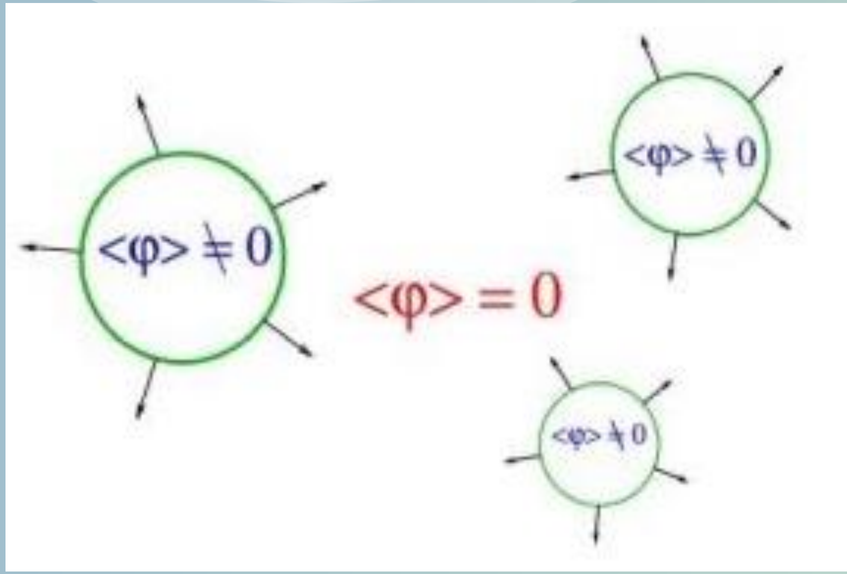

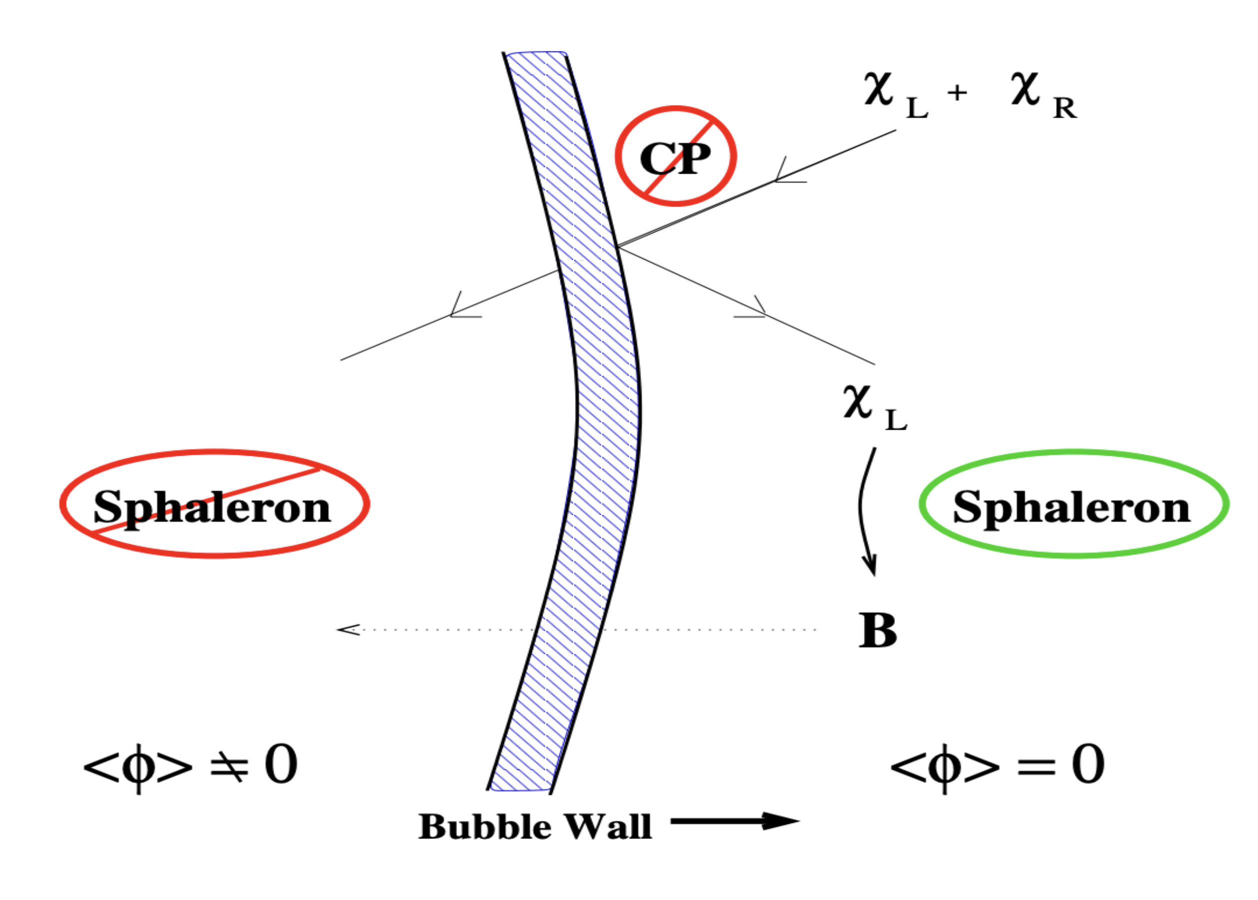

Morrissey & Ramsey-Musolf [1206.2942]

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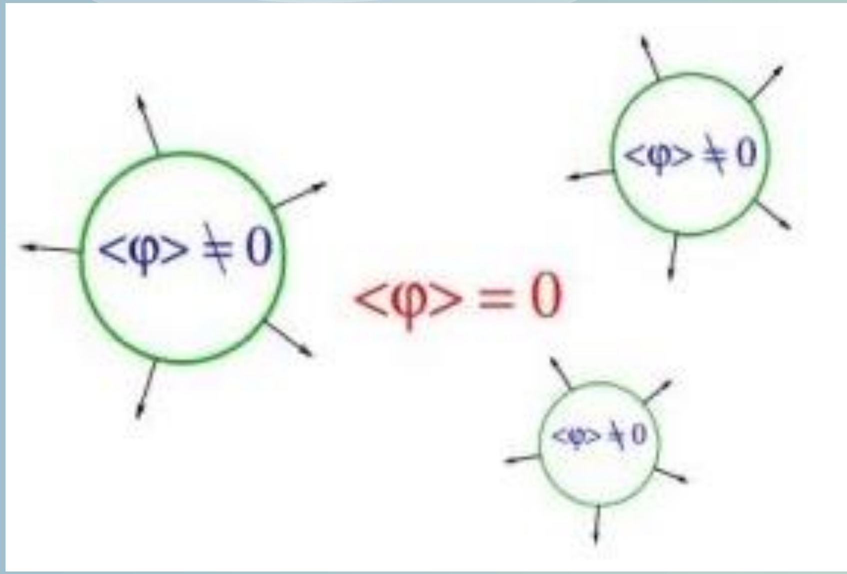
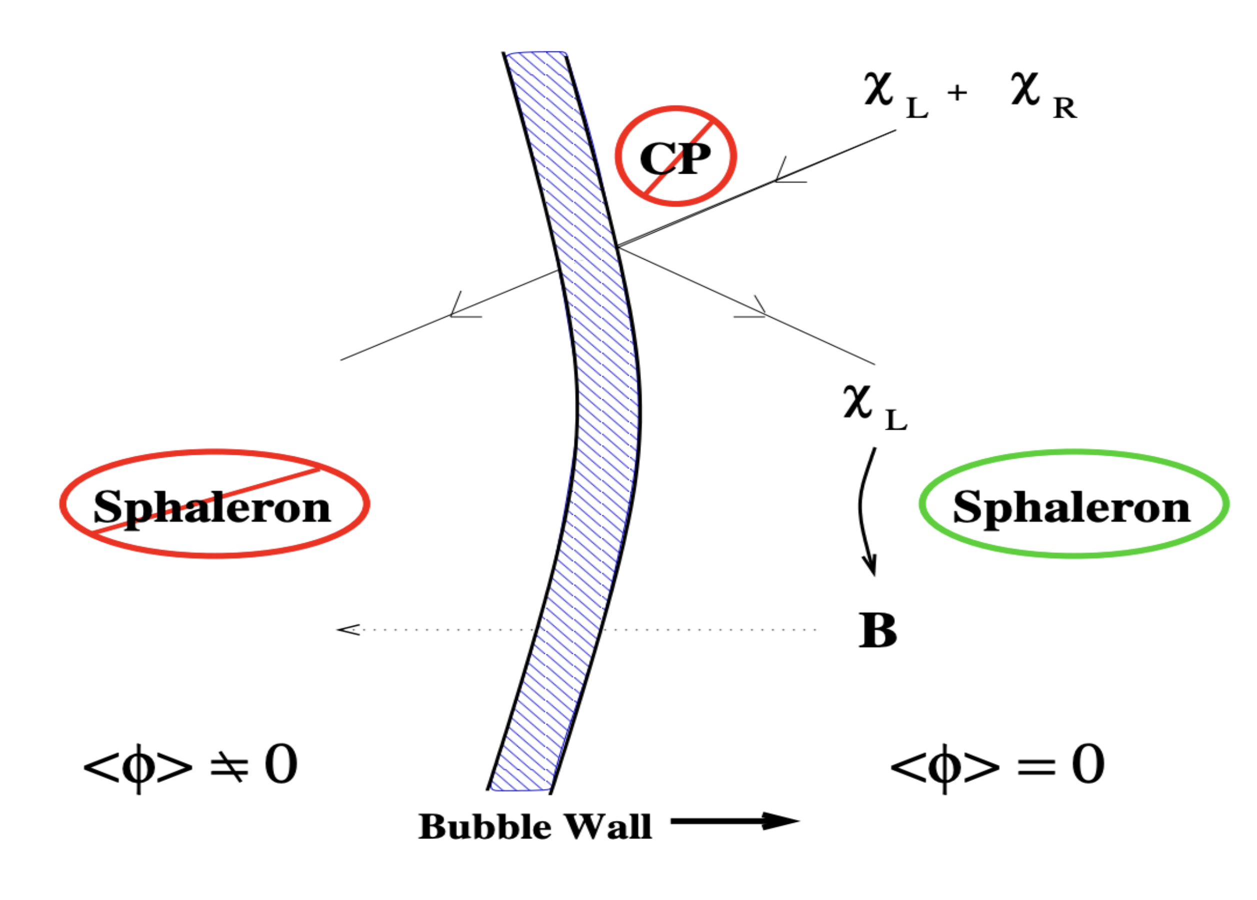


EW sphalerons → Baryon number violation



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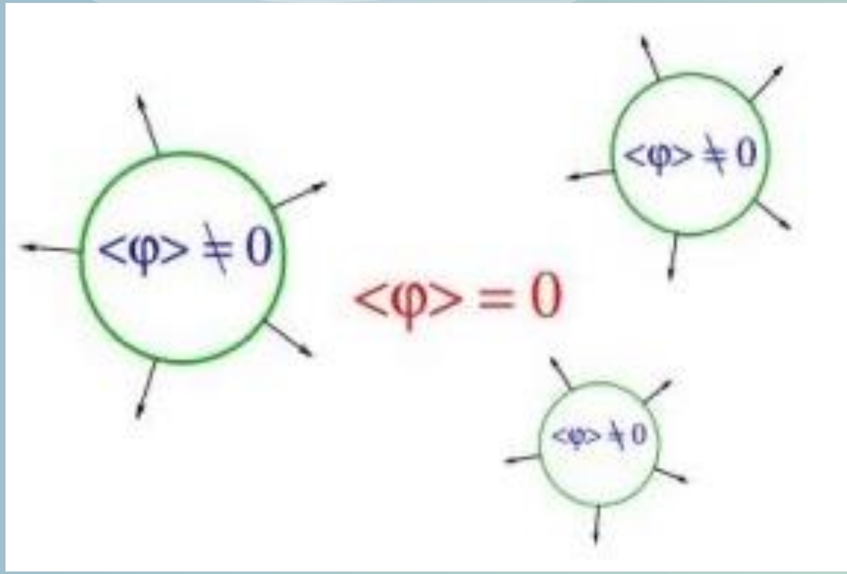

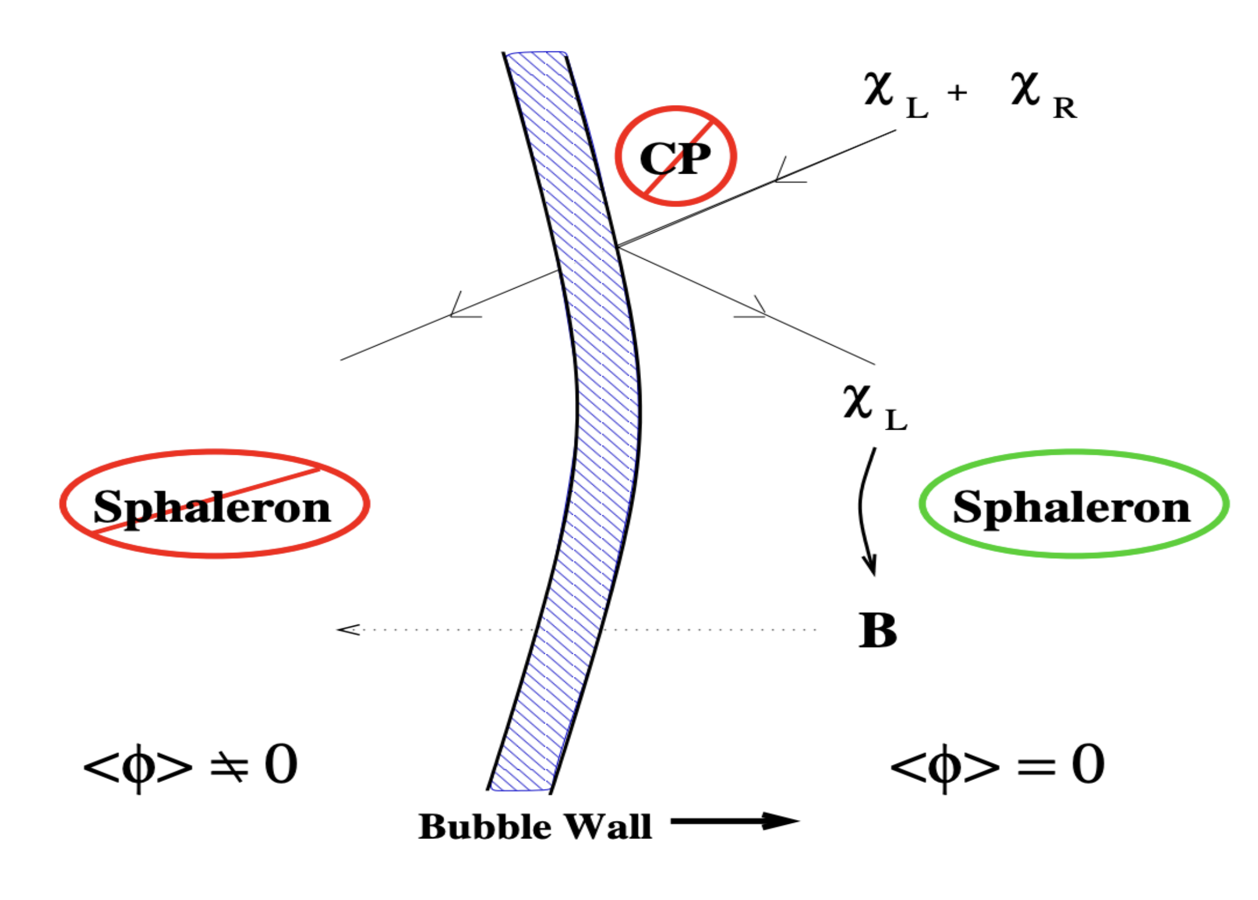

EW sphalerons → Baryon number violation

CKM matrix (or BSM physics) → C and CP violation

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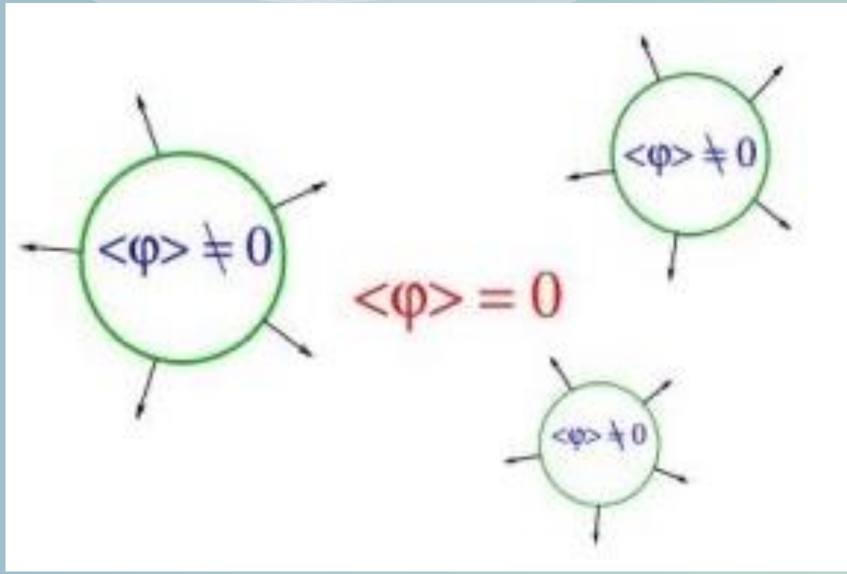

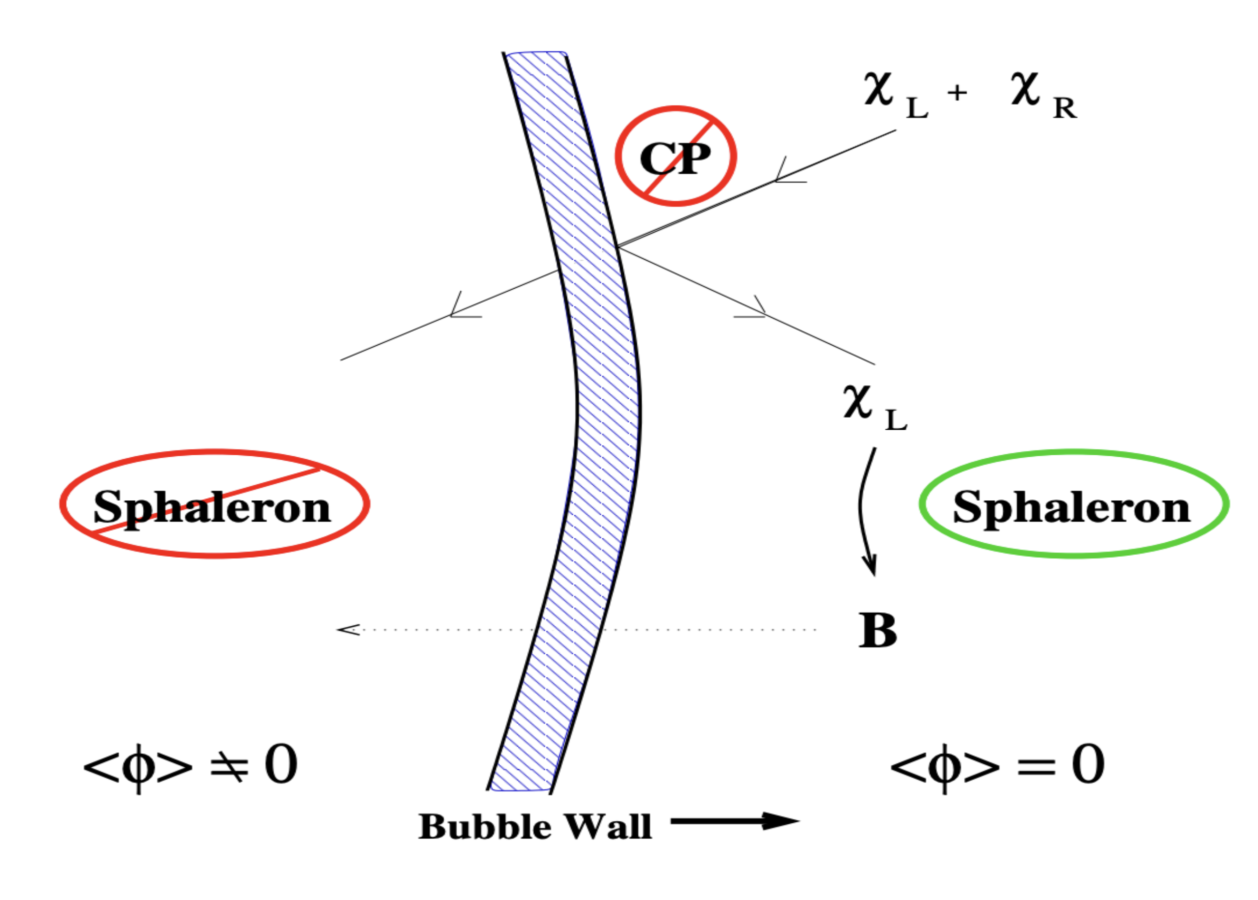

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Bubble wall motion → departure from thermal equilibrium

Morrissey & Ramsey-Musolf [1206.2942]

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$10^{-16}G < B < 10^{-9}G$ on Mpc scales
(lower bounds from blazars and upper from CMB)

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EW Magnetogenesis: Kibble Mechanism

EWSSB → $|\phi|^2 = \phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2 = \eta^2$

Higgs takes different values in causally disconnected zones
→ Vacuum Manifold $S^2 \times S^1$

[2010.10525, 2108.05357, 2302.00512]

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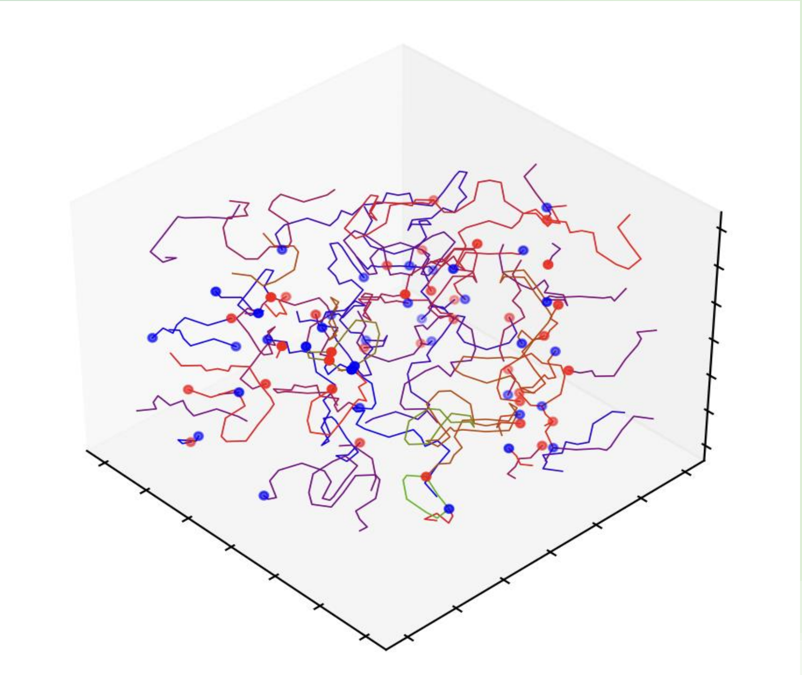

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Monopoles and Strings → $\vec{\nabla} \cdot \vec{B} \neq 0$

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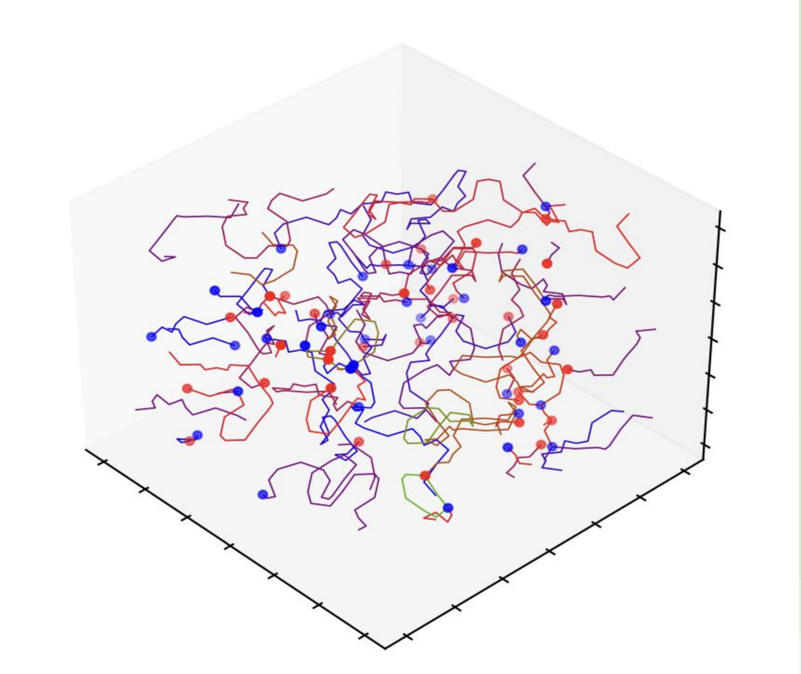


EW Magnetogenesis: Kibble Mechanism

EWSSB → $|\phi|^2 = \phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2 = \eta^2$

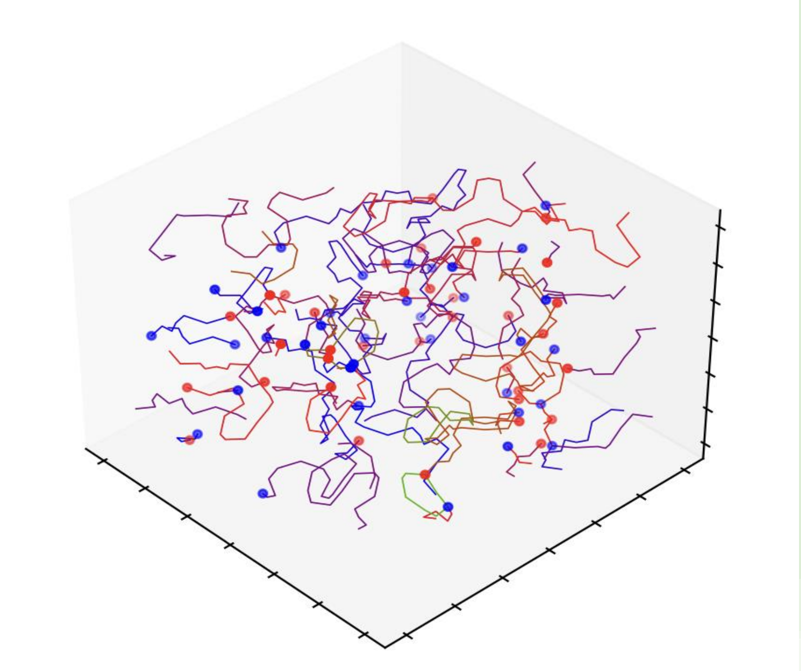
Higgs takes different values in causally disconnected zones
→ Vacuum Manifold $S^2 \times S^1$

Monopoles and Strings → $\vec{\nabla} \cdot \vec{B} \neq 0$

'‘t Hooft, Vachaspati *et al.* → $A_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i\frac{2\sin\theta_w}{g}(\partial_\mu \hat{\Phi}^\dagger \partial_\nu \hat{\Phi} - \partial_\nu \hat{\Phi}^\dagger \partial_\mu \hat{\Phi})$

[Nucl. Phys. B 79 (1974) 276, 2010.10525, 2108.05357, 2302.00512]

Introduction: phase transitions and primordial magnetic fields



$10^{-16}G < B < 10^{-9}G$ on Mpc scales
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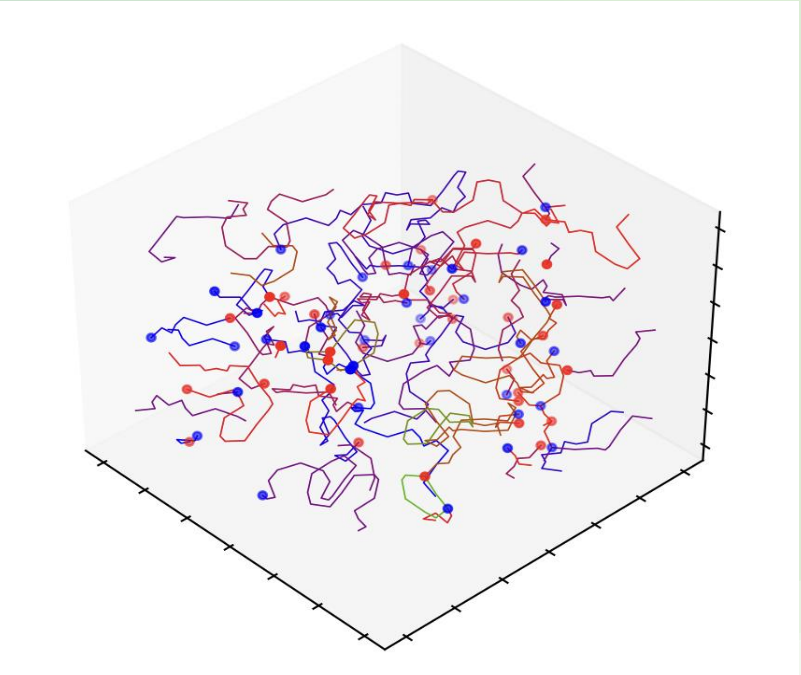
Annihilation of monopoles-antimonopoles pairs with residual $\vec{B} \neq 0$

[Nucl. Phys. B 79 (1974) 276, 2010.10525, 2108.05357, 2302.00512]

Introduction: *first-order* phase transitions and primordial magnetic fields



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EW Magnetogenesis: Kibble Mechanism

EWSSB → $|\phi|^2 = \phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2 = \eta^2$

Higgs takes different values in *different broken phase bubbles*
→ Vacuum Manifold $S^2 \times S^1$

Monopoles and Strings → $\vec{\nabla} \cdot \vec{B} \neq 0$

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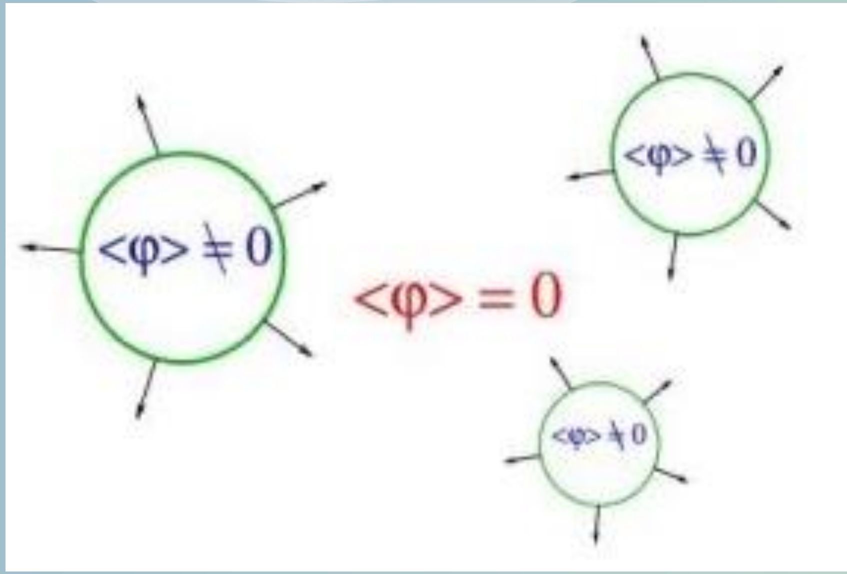
Introduction: first-order phase transitions and gravitational waves

First-Order Phase Transitions occur through the nucleation of broken phase bubbles



Introduction: first-order phase transitions and gravitational waves

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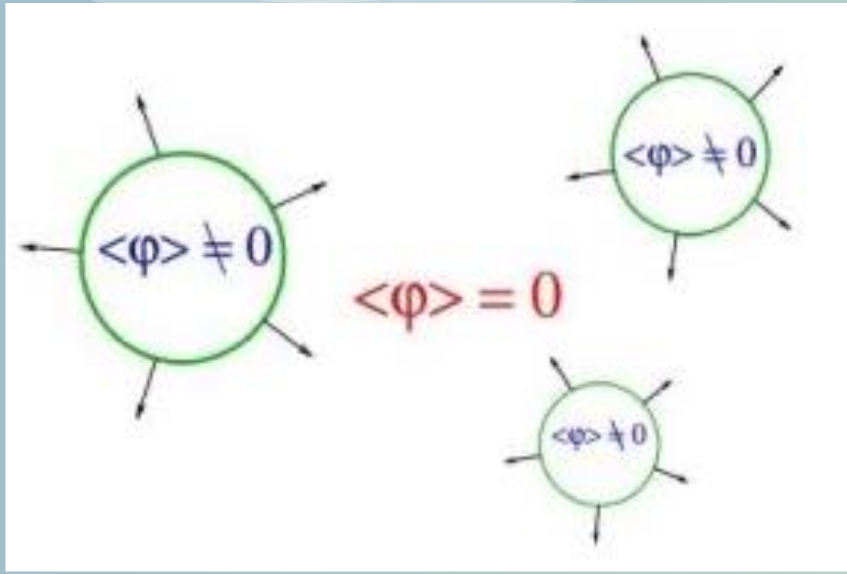
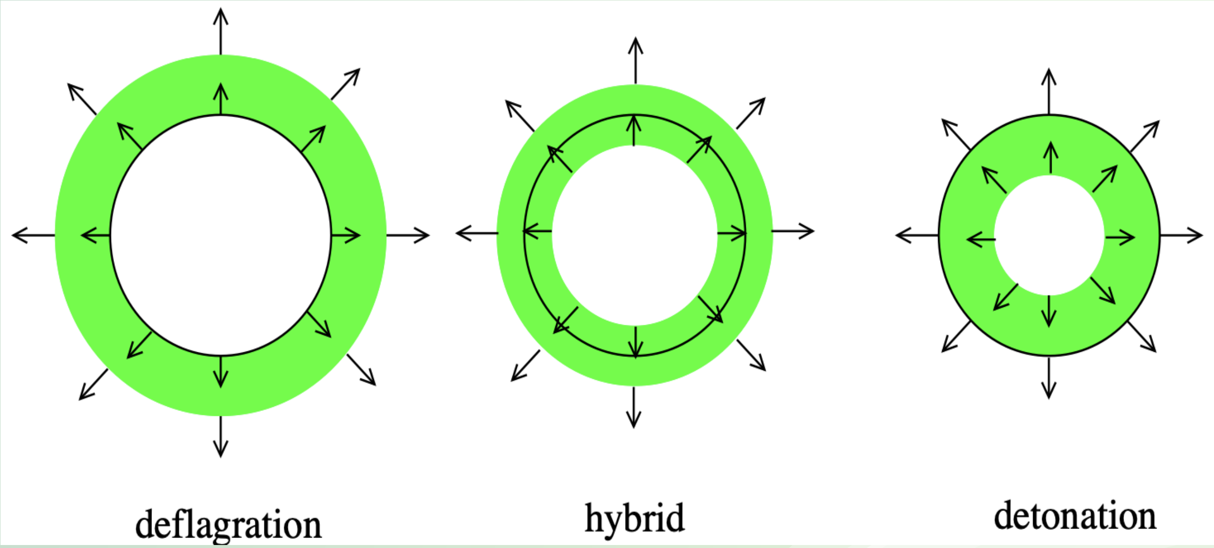


friction between scalar and plasma



Espinosa et al. [1004.4187]

Introduction: first-order phase transitions and gravitational waves

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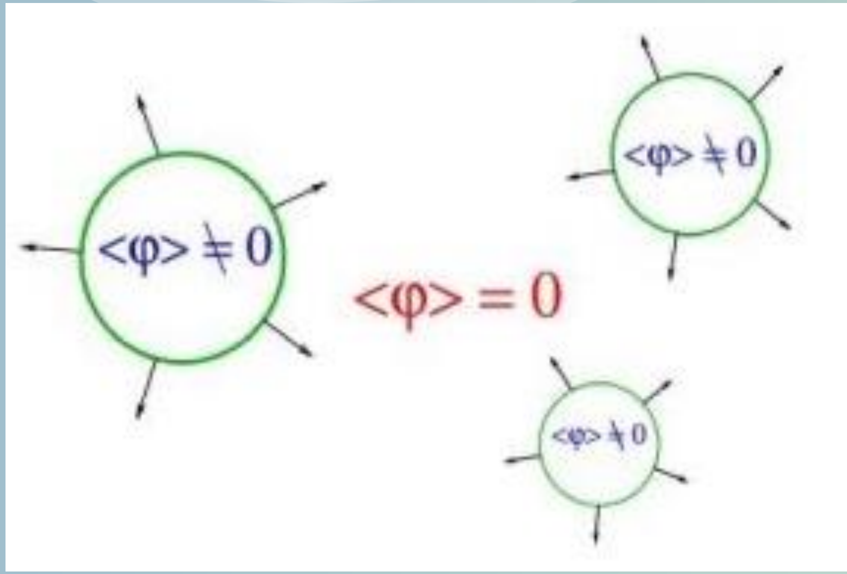

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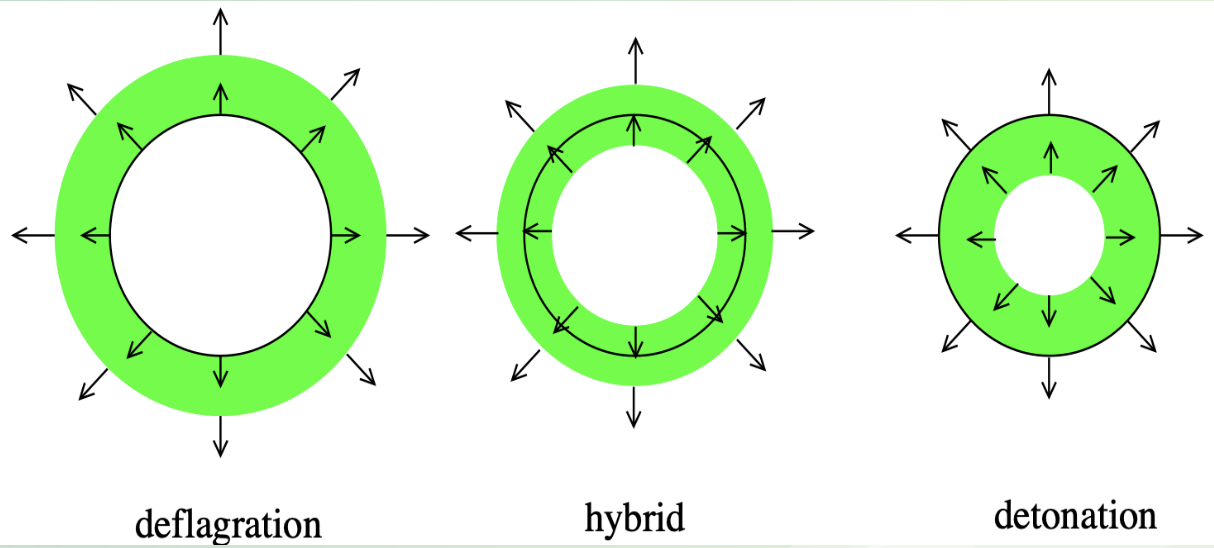

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Bubble expansion phase → scalar and fluid profiles are spherically symmetric

Introduction: first-order phase transitions and gravitational waves

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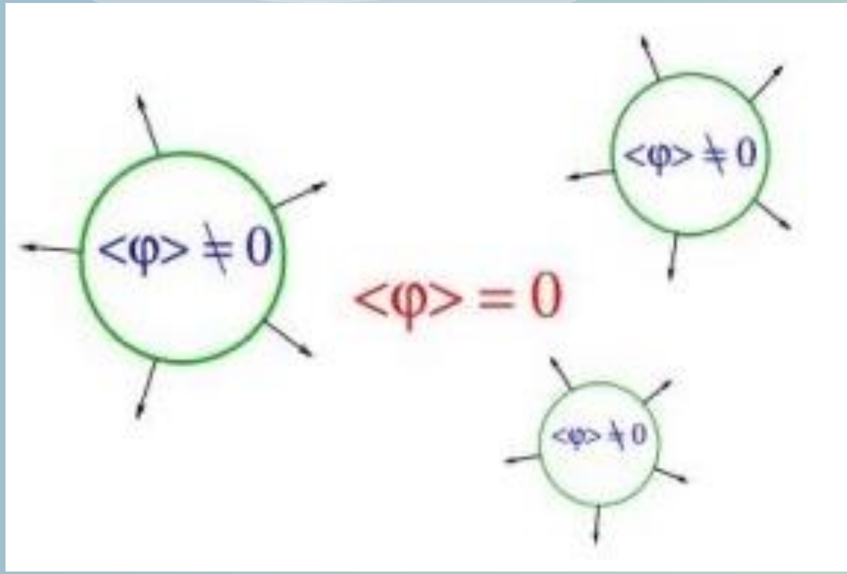

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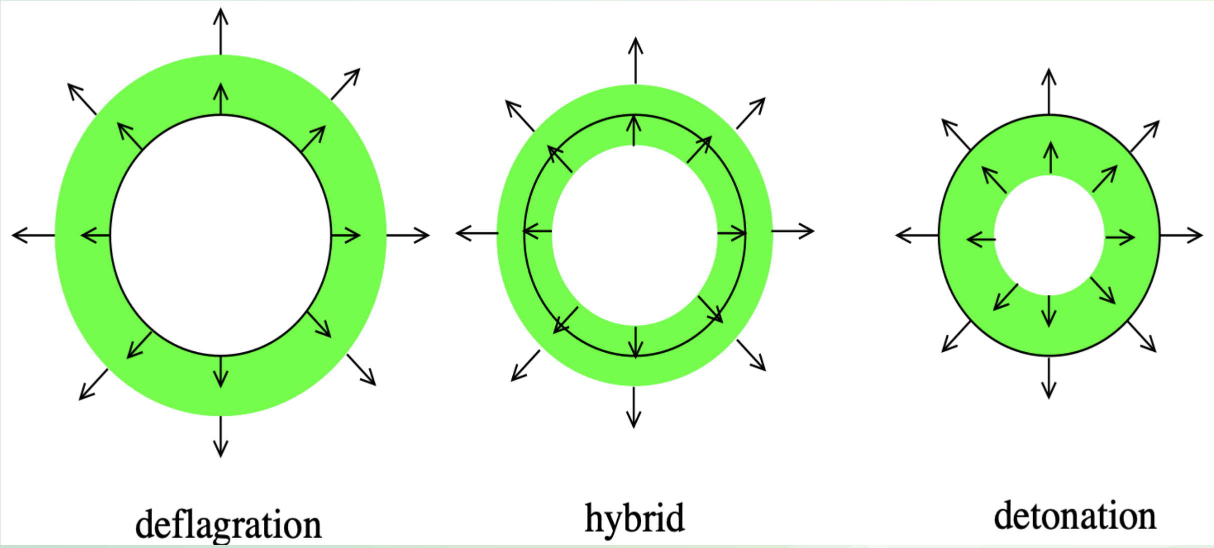


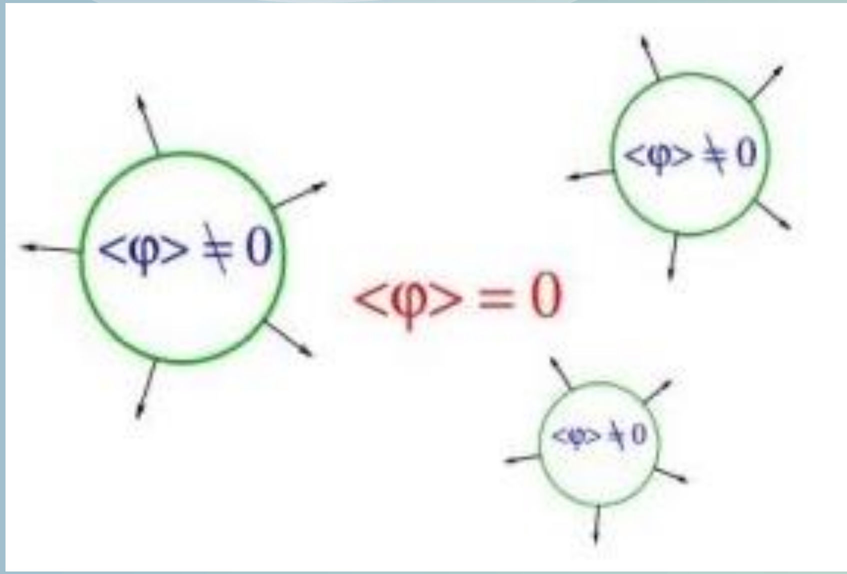
Espinosa et al. [1004.4187]

Bubble expansion phase → scalar and fluid profiles are spherically symmetric

No anisotropic stresses → No gravitational wave production (see Lecture by Chiara)

Introduction: first-order phase transitions and gravitational waves

First-Order Phase Transitions occur through the nucleation of broken phase bubbles



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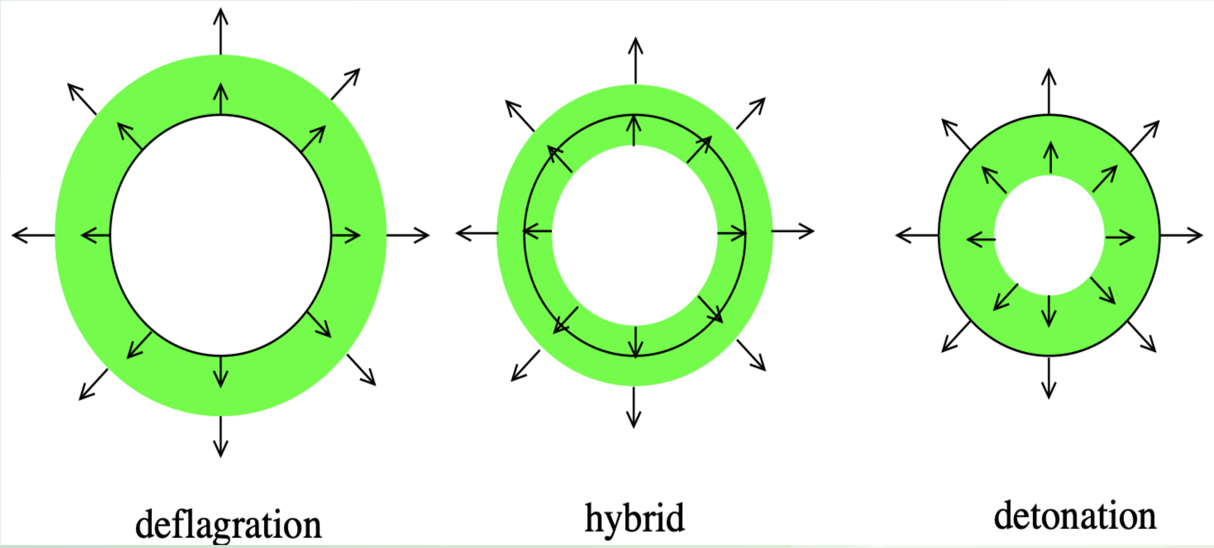

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Bubble collisions break spherical symmetry

Introduction: first-order phase transitions and gravitational waves

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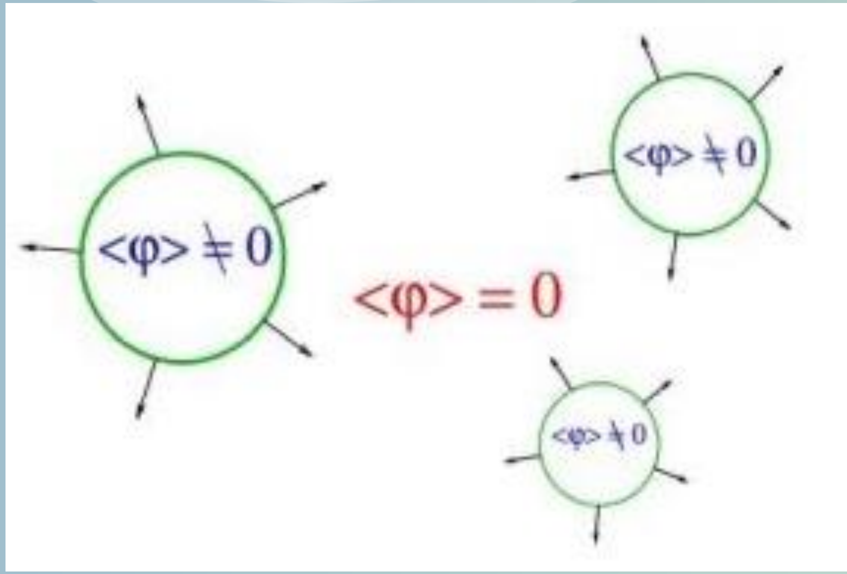

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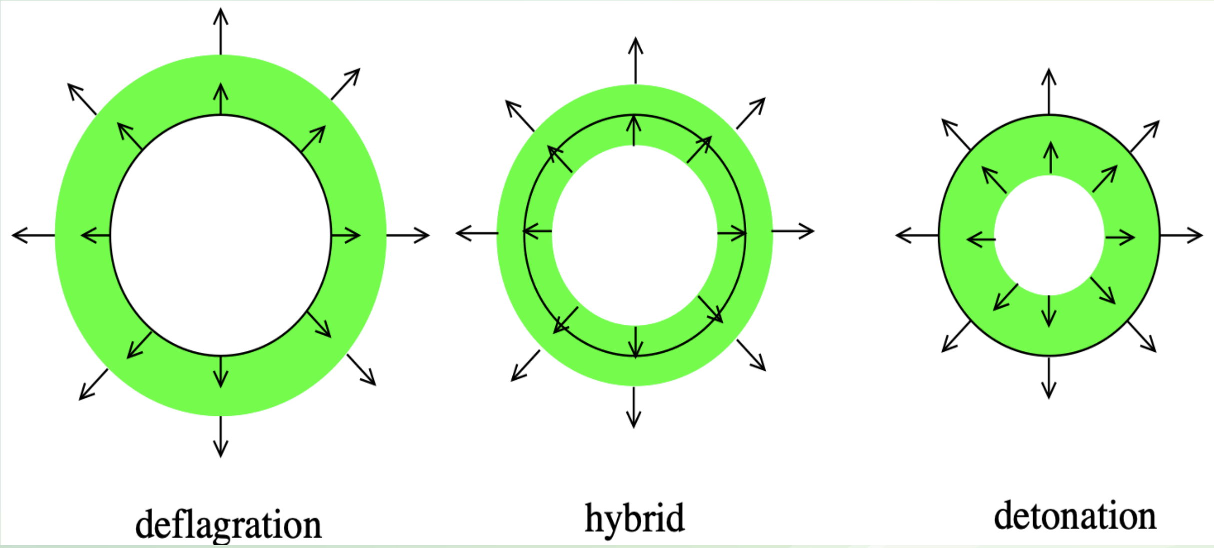
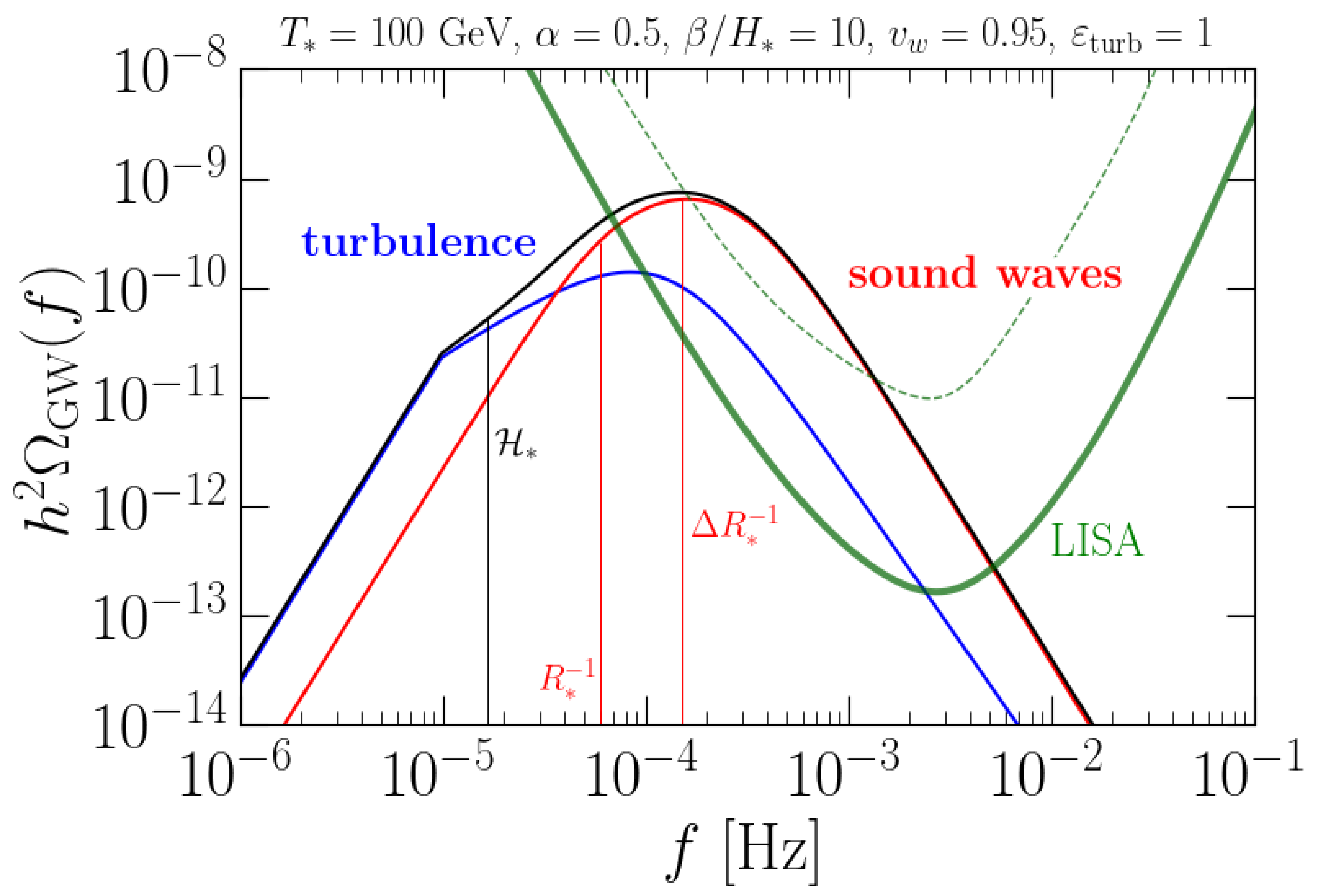


Espinosa et al. [1004.4187]

Bubble collisions break spherical symmetry

Nonzero anisotropic stresses → scalar and fluid can produce gravitational waves

Introduction: first-order phase transitions and gravitational waves

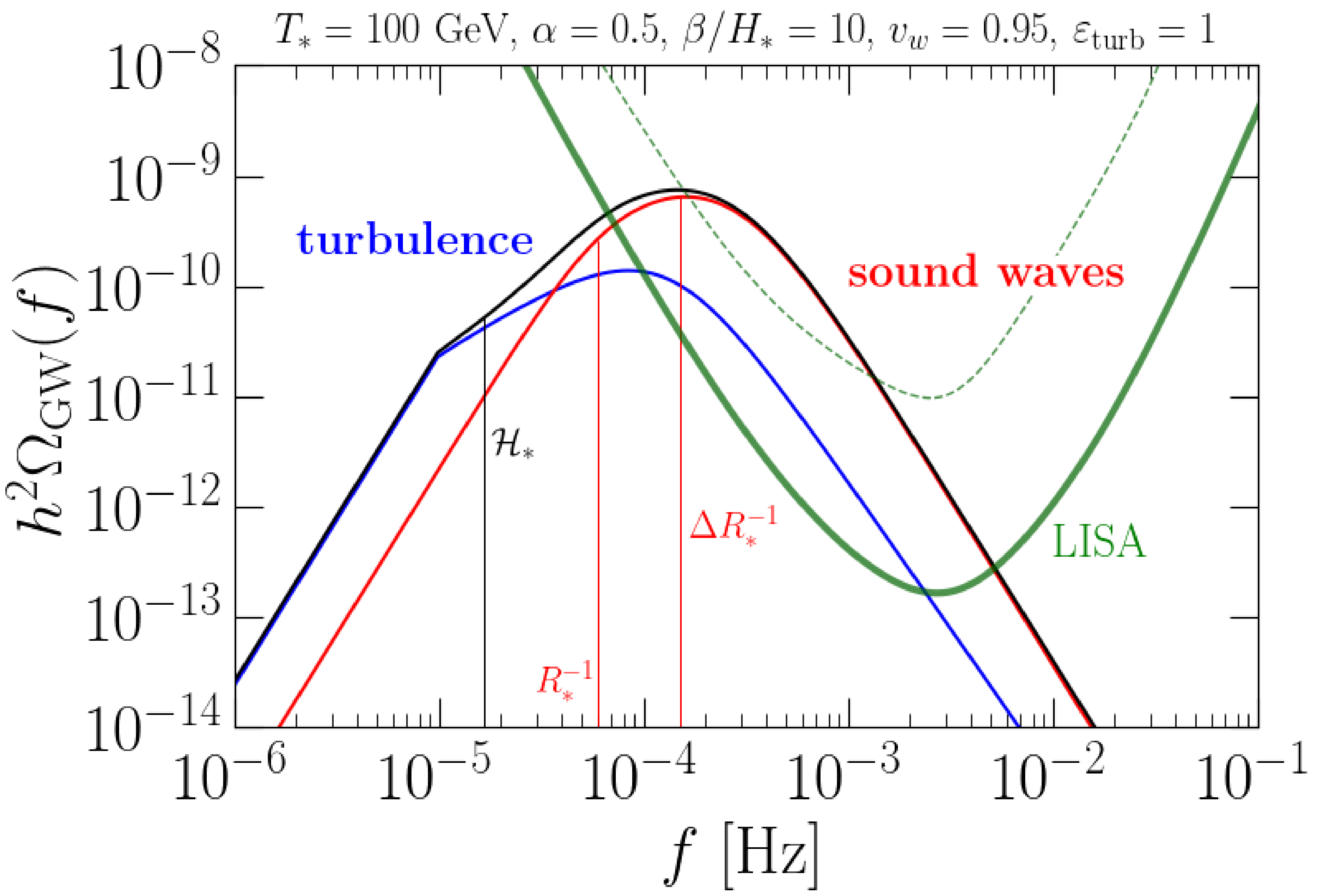


GW background from EW phase transition in the LISA sensitivity band!

← Credits: Alberto Roper Pol

Lisa Cosmology Working Group [2403.03723]

Introduction: first-order phase transitions and gravitational waves



Sound-shell model

Hindmarsh & Hijazi [1909.10040]

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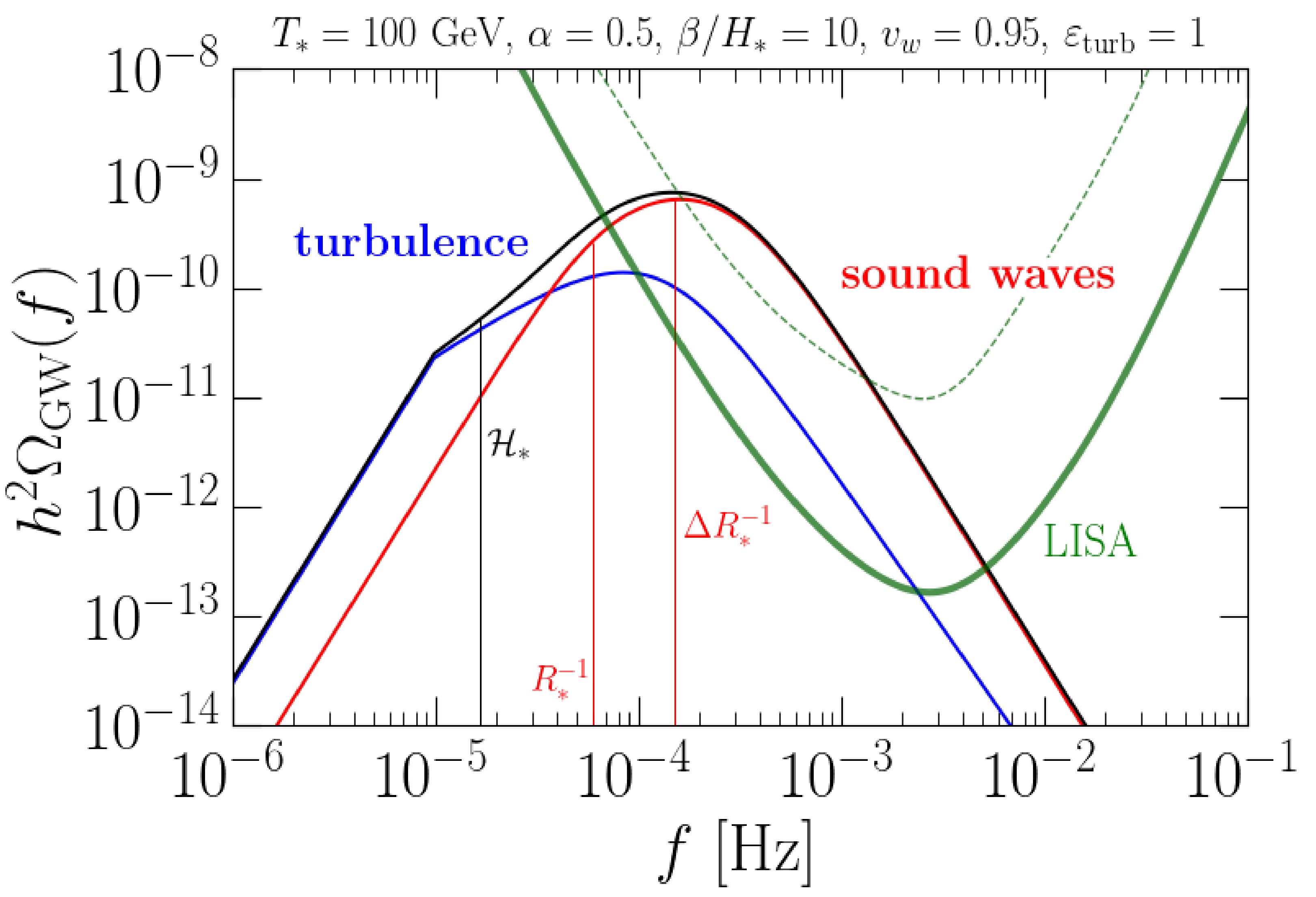

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Constant-in-time model

Roper Pol, Caprini et al. [2201.05630]

GW background from EW phase transition in the LISA sensitivity band!

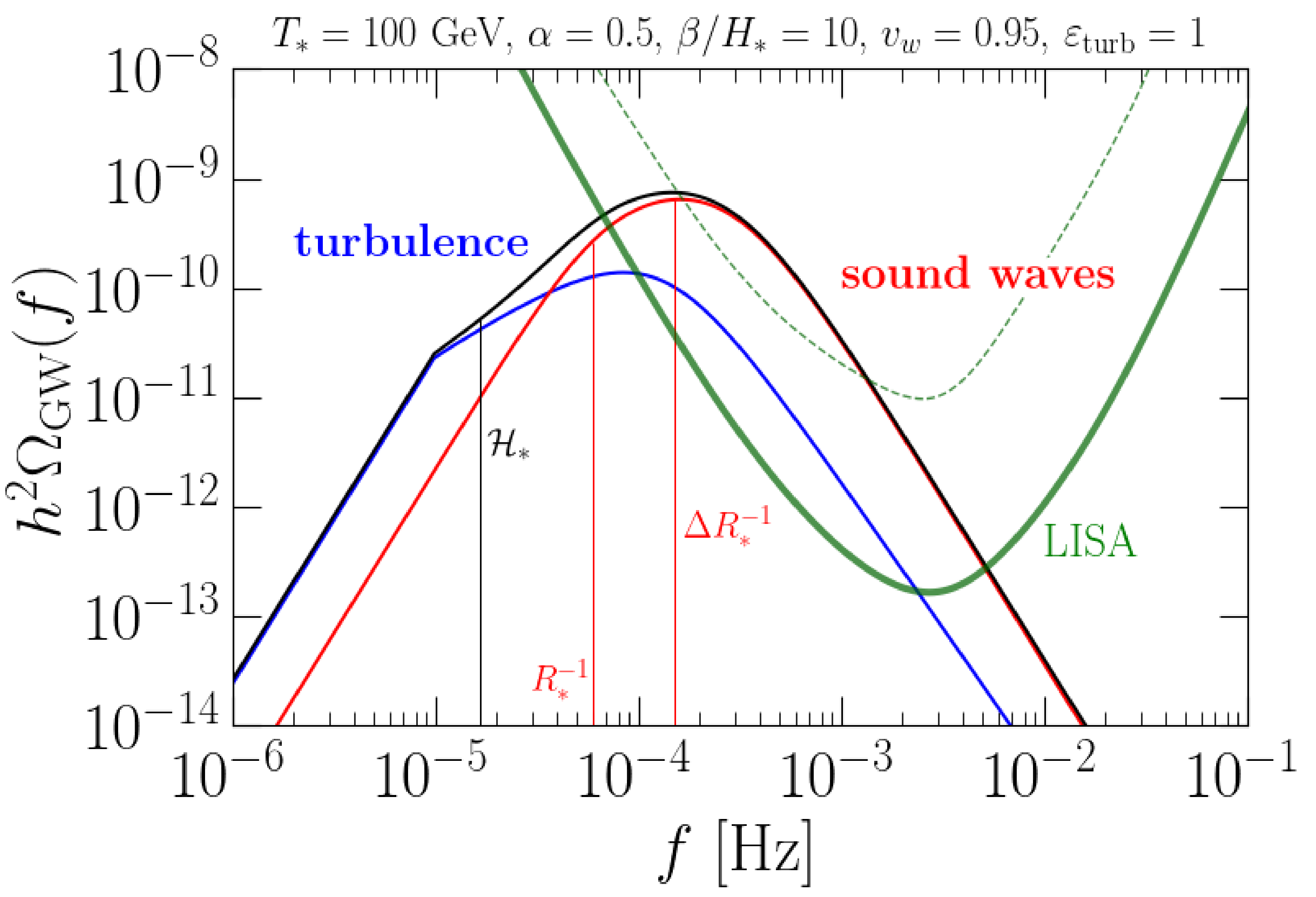
← Credits: Alberto Roper Pol

Lisa Cosmology Working Group [2403.03723]

Gravitational Waves from sound waves

[*Ongoing work* in collaboration with C. Caprini, S. Procacci, A. Roper Pol]

Introduction: first-order phase transitions and gravitational waves



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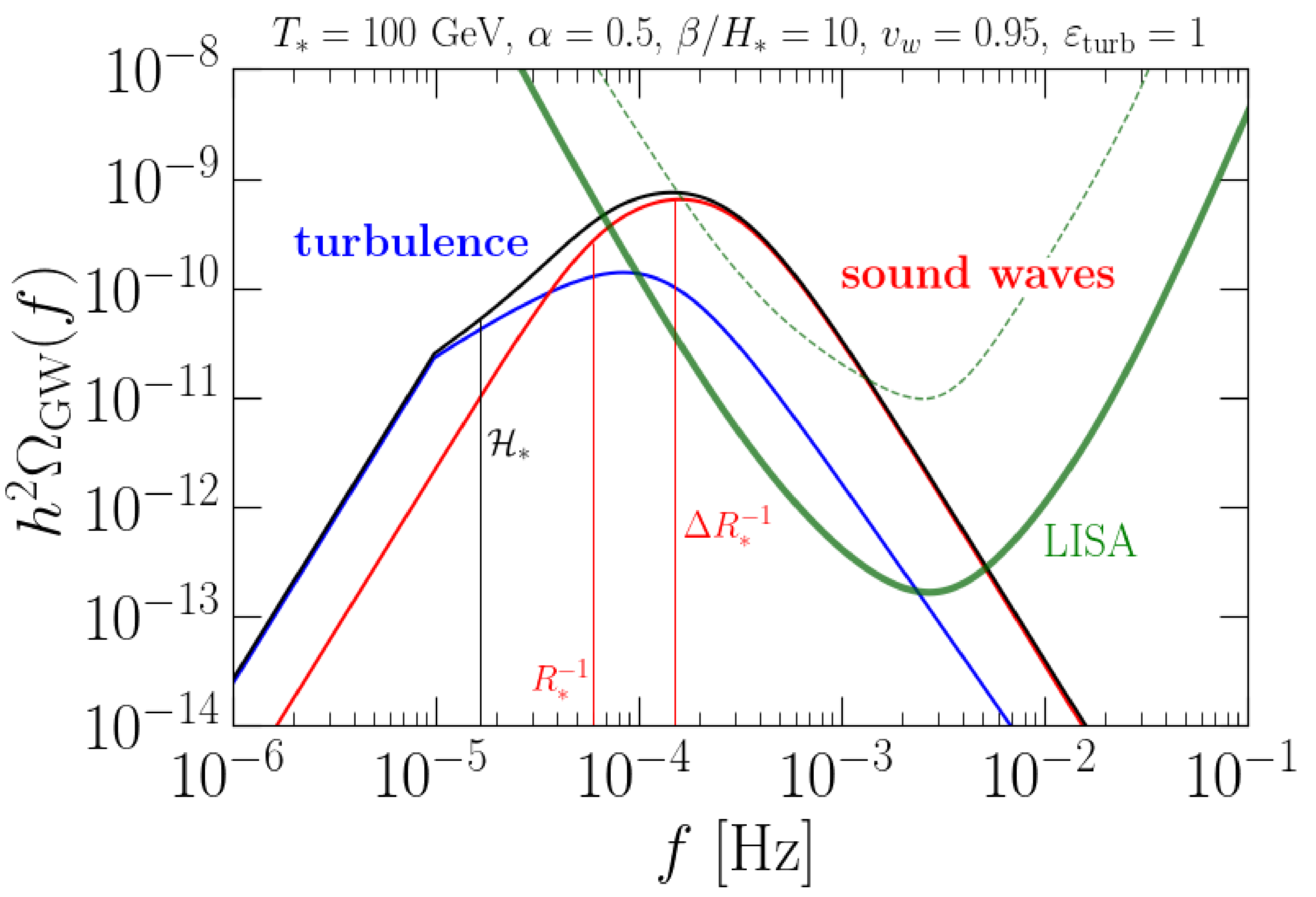

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Hindmarsh & Hijazi [1909.10040]

What is the origin of the peak scales in the GW spectrum from sound waves?

Are they actually related to R_* & ΔR_*?

← Credits: Alberto Roper Pol

Lisa Cosmology Working Group [2403.03723]

Fluid perturbations from expanding scalar bubbles

$$T^{tot}_{\mu\nu} = w_{tot}\, u_\mu u_\nu + p_{tot}\, g_{\mu\nu} + \partial_\mu\phi\, \partial_\nu\phi - g_{\mu\nu}\left(\frac{1}{2}\partial_\sigma\phi\, \partial^\sigma\phi\right)$$

$$w_{tot} = w - T\frac{\partial V_{eff}(\phi, T)}{\partial T}$$

$$p_{tot} = p - V_{eff}(\phi, T)$$

Fluid perturbations from expanding scalar bubbles

$$T_{\mu\nu}^{tot} = w_{tot}\, u_\mu u_\nu + p_{tot}\, g_{\mu\nu} + \partial_\mu\phi\, \partial_\nu\phi - g_{\mu\nu}\left(\frac{1}{2}\partial_\sigma\phi\, \partial^\sigma\phi\right)$$

$$w_{tot} = w - T\frac{\partial V_{eff}(\phi, T)}{\partial T}$$

$$p_{tot} = p - V_{eff}(\phi, T)$$

$$\begin{cases} \nabla_\mu T_{tot}^{\mu\nu} = 0 \\ \nabla_\sigma(\partial^\sigma\phi) - \dfrac{\partial V}{\partial\phi} = \delta_{friction} \end{cases}$$

$\eta\, u^\mu \partial_\mu \phi$?

Fluid perturbations from expanding scalar bubbles

$$T^{tot}_{\mu\nu} = w_{tot}\, u_\mu u_\nu + p_{tot}\, g_{\mu\nu} + \partial_\mu\phi\, \partial_\nu\phi - g_{\mu\nu}\left(\frac{1}{2}\partial_\sigma\phi\, \partial^\sigma\phi\right)$$

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$\eta\, u^\mu \partial_\mu \phi$?

Full picture requires lattice simulations

[1504.03291] [2409.03651][2505.17824]

What can we understand analytically?

Fluid perturbations from expanding scalar bubbles

$$T_{\mu\nu}^{tot} = w_{tot}\, u_\mu u_\nu + p_{tot}\, g_{\mu\nu} + \partial_\mu\phi\, \partial_\nu\phi - g_{\mu\nu}\left(\frac{1}{2}\partial_\sigma\phi\, \partial^\sigma\phi\right)$$

Simplifying assumptions:

Fluid perturbations from expanding scalar bubbles

$$T_{\mu\nu}^{tot} = w_{tot}\, u_\mu u_\nu + p_{tot}\, g_{\mu\nu} + \partial_\mu \phi\, \partial_\nu \phi - g_{\mu\nu} \left(\frac{1}{2} \partial_\sigma \phi\, \partial^\sigma \phi \right)$$

Simplifying assumptions:

- Flat spacetime $g_{\mu\nu} \to \eta_{\mu\nu}$

Fluid perturbations from expanding scalar bubbles

$$T^{tot}_{\mu\nu} = w_{tot}\, u_\mu u_\nu + p_{tot}\, g_{\mu\nu} + \partial_\mu\phi\, \partial_\nu\phi - g_{\mu\nu}\left(\frac{1}{2}\partial_\sigma\phi\, \partial^\sigma\phi\right)$$

Simplifying assumptions:

- Flat spacetime $g_{\mu\nu} \to \eta_{\mu\nu}$
- Bag equation of state → (+) Symmetric phase (−) Broken phase →

$$p^{\pm}_{tot} = \frac{1}{3}a_{\pm}T^4_{\pm} - \epsilon_{\pm}$$

$$e^{\pm}_{tot} = a_{\pm}T^4_{\pm} + \epsilon_{\pm}$$

$$w^{\pm}_{tot} = e^{\pm}_{tot} + p^{\pm}_{tot}$$

Fluid perturbations from expanding scalar bubbles

$$T^{tot}_{\mu\nu} = w_{tot}\, u_\mu u_\nu + p_{tot}\, g_{\mu\nu} + \partial_\mu\phi\, \partial_\nu\phi - g_{\mu\nu}\left(\frac{1}{2}\partial_\sigma\phi\, \partial^\sigma\phi\right)$$

Simplifying assumptions:

- Flat spacetime $g_{\mu\nu} \to \eta_{\mu\nu}$
- Bag equation of state → (+) Symmetric phase, (−) Broken phase →
- Neglect scalar field profiles

$$p^{\pm}_{tot} = \frac{1}{3} a_\pm T^4_\pm - \epsilon_\pm$$

$$e^{\pm}_{tot} = a_\pm T^4_\pm + \epsilon_\pm$$

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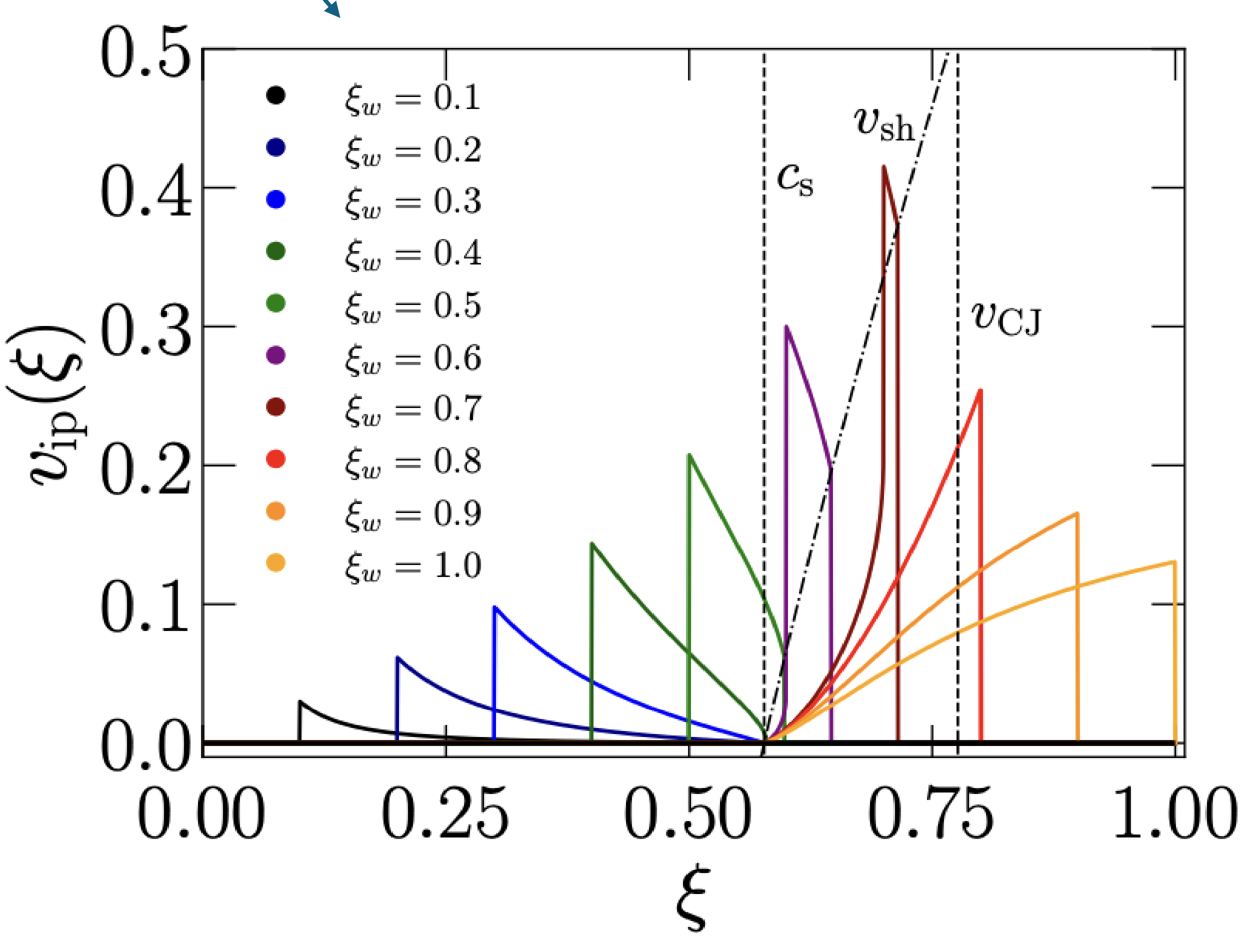
Fluid perturbations from expanding scalar bubbles

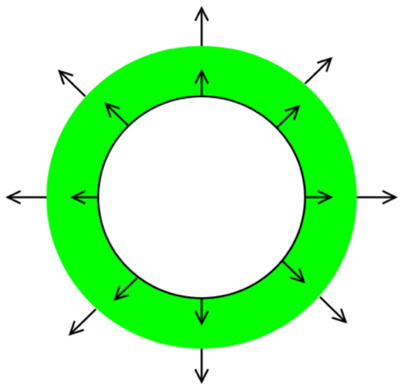
pip install cosmoGW



Fluid perturbations from expanding scalar bubbles

DEFLAGRATIONS

$\xi_w < c_s$



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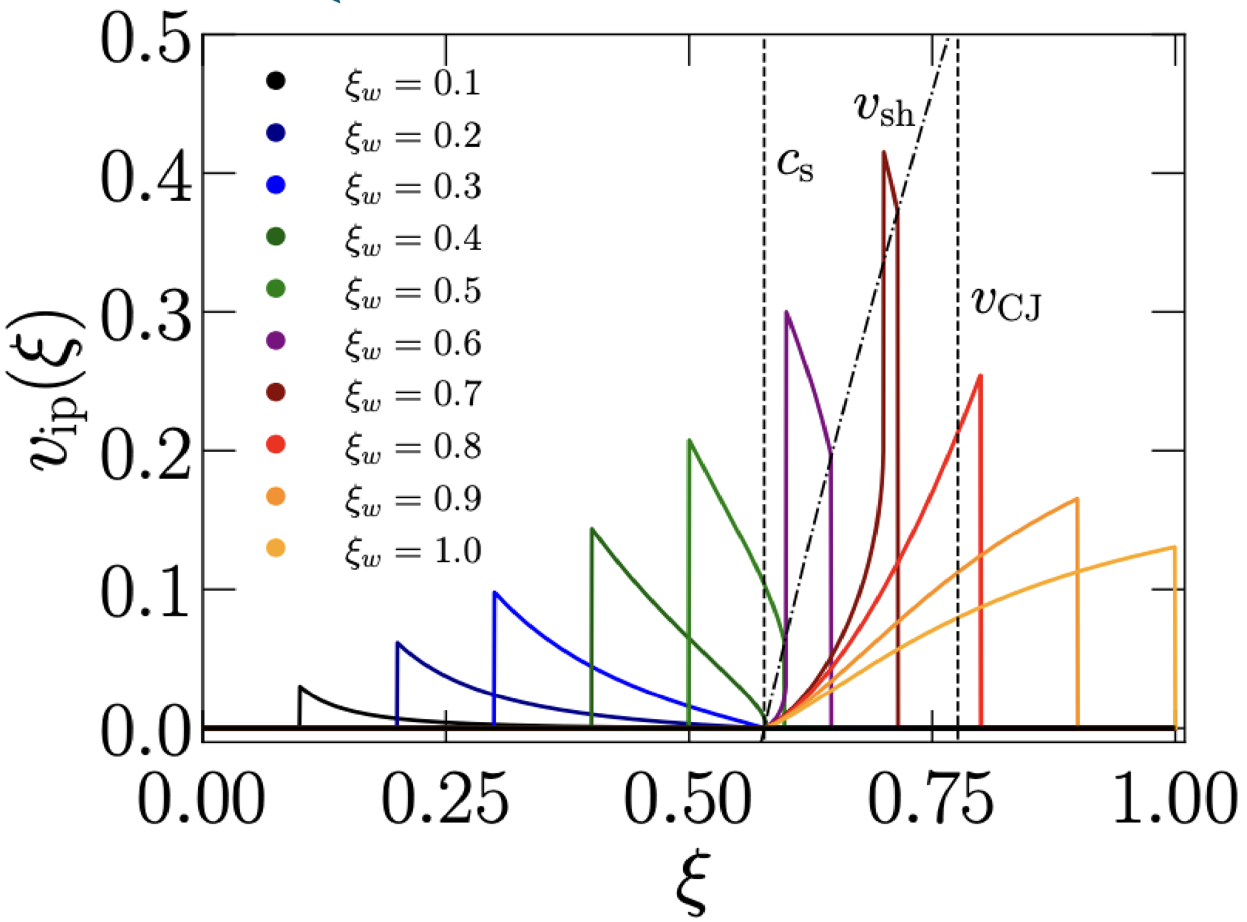

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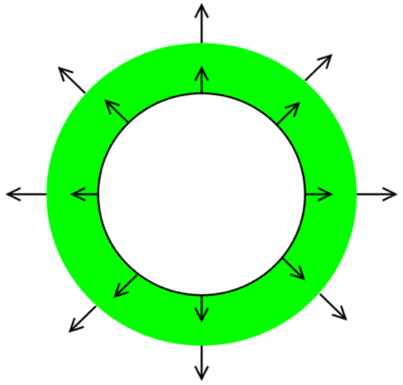
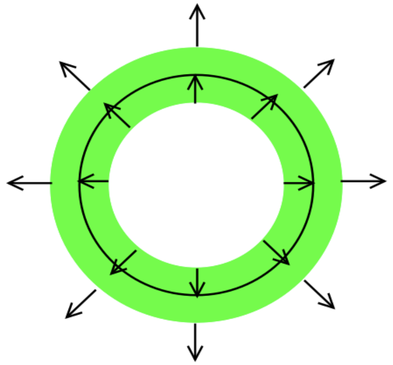


HYBRIDS

$c_s < \xi_w < v_{CJ}(\alpha)$



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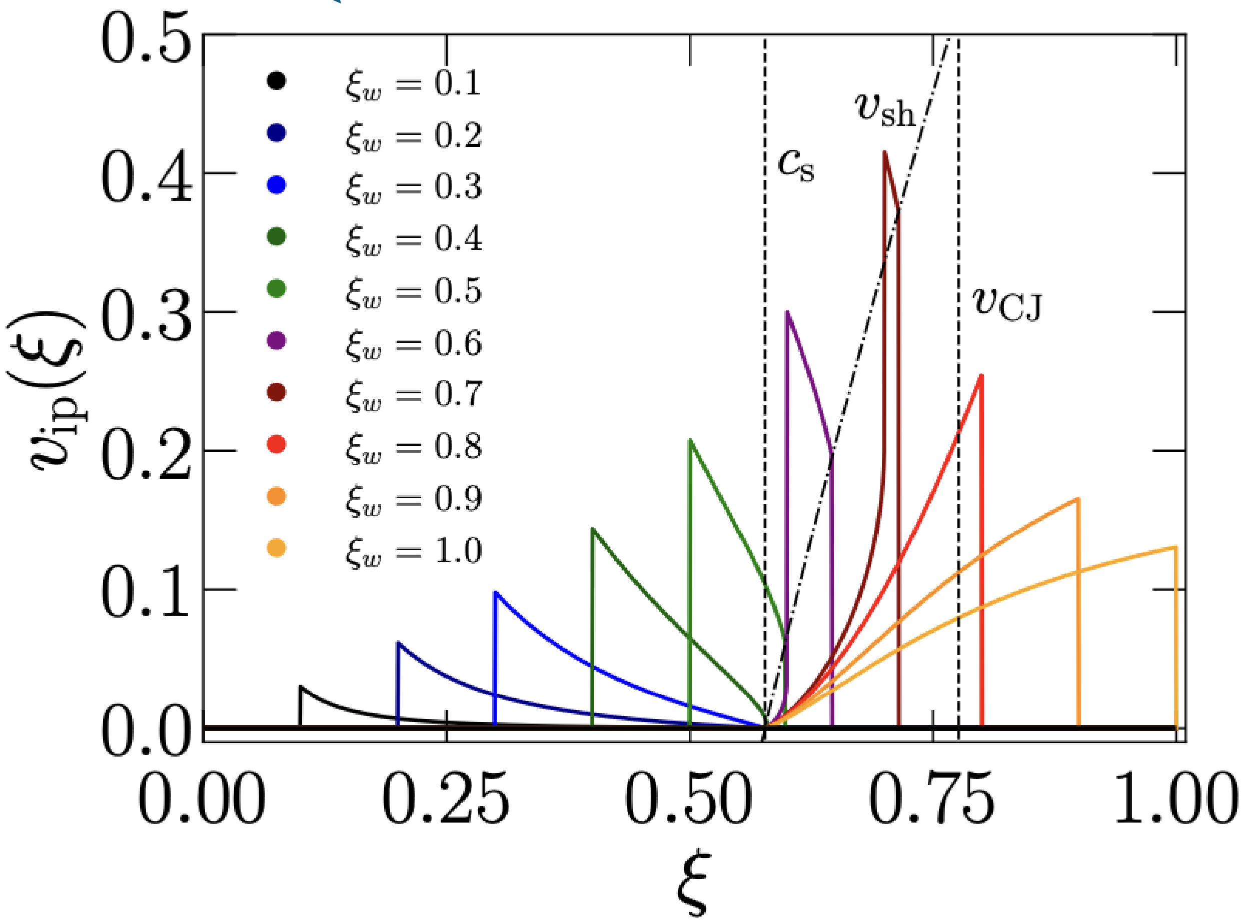
$$v_{CJ}(\alpha) = \frac{1 + \sqrt{\alpha(2 + 3\alpha)}}{\sqrt{3}(1 + \alpha)}$$



Fluid perturbations from expanding scalar bubbles

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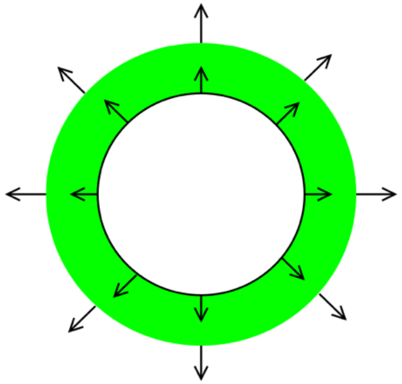

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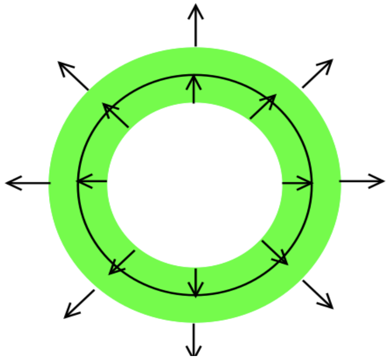
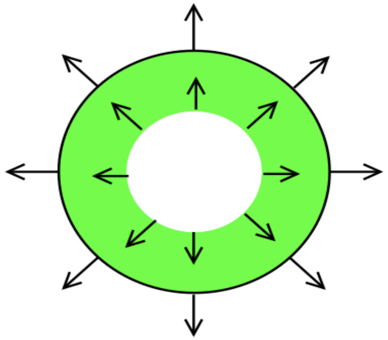


DETONATIONS

$\xi_w > v_{CJ}(\alpha)$



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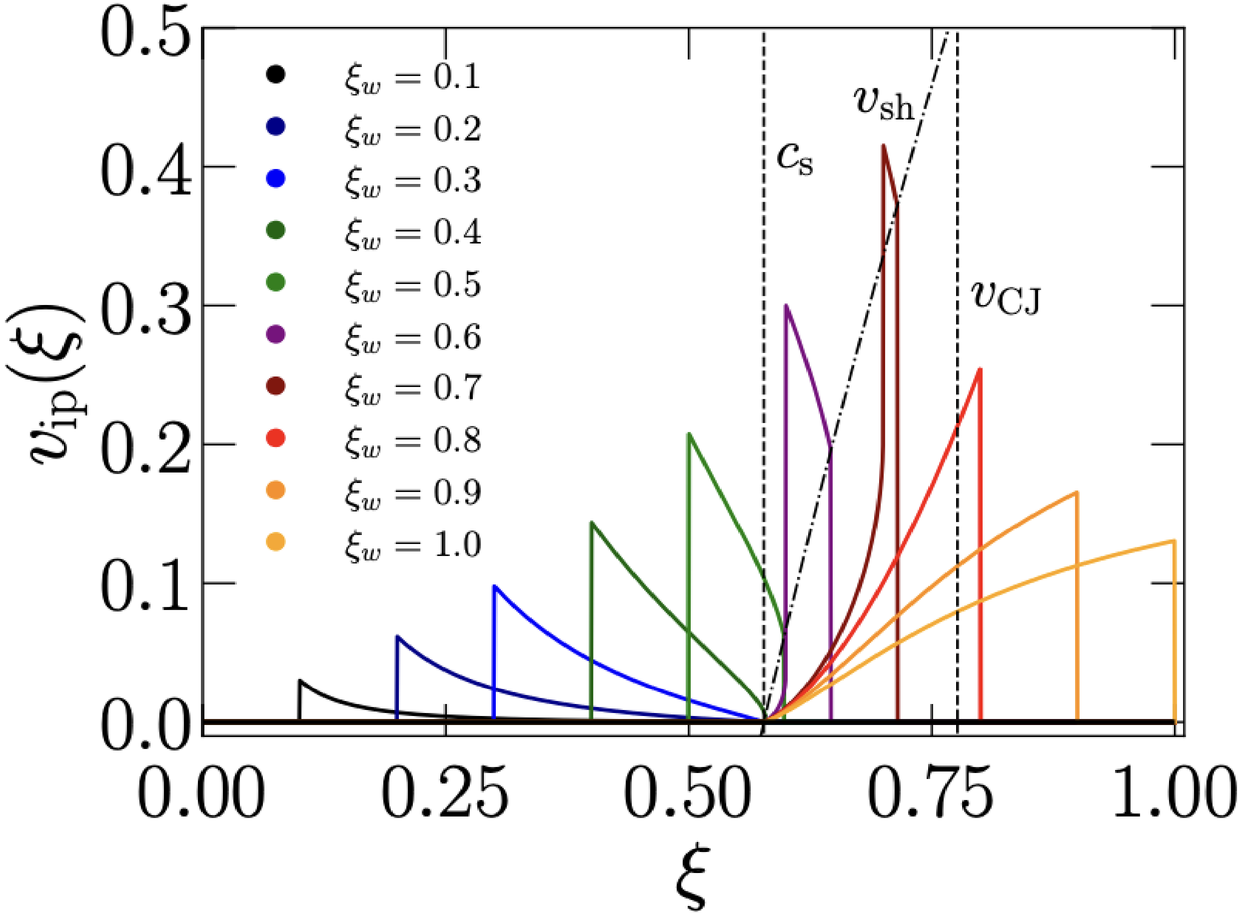


Fluid perturbations from expanding scalar bubbles

Properties of the profiles:

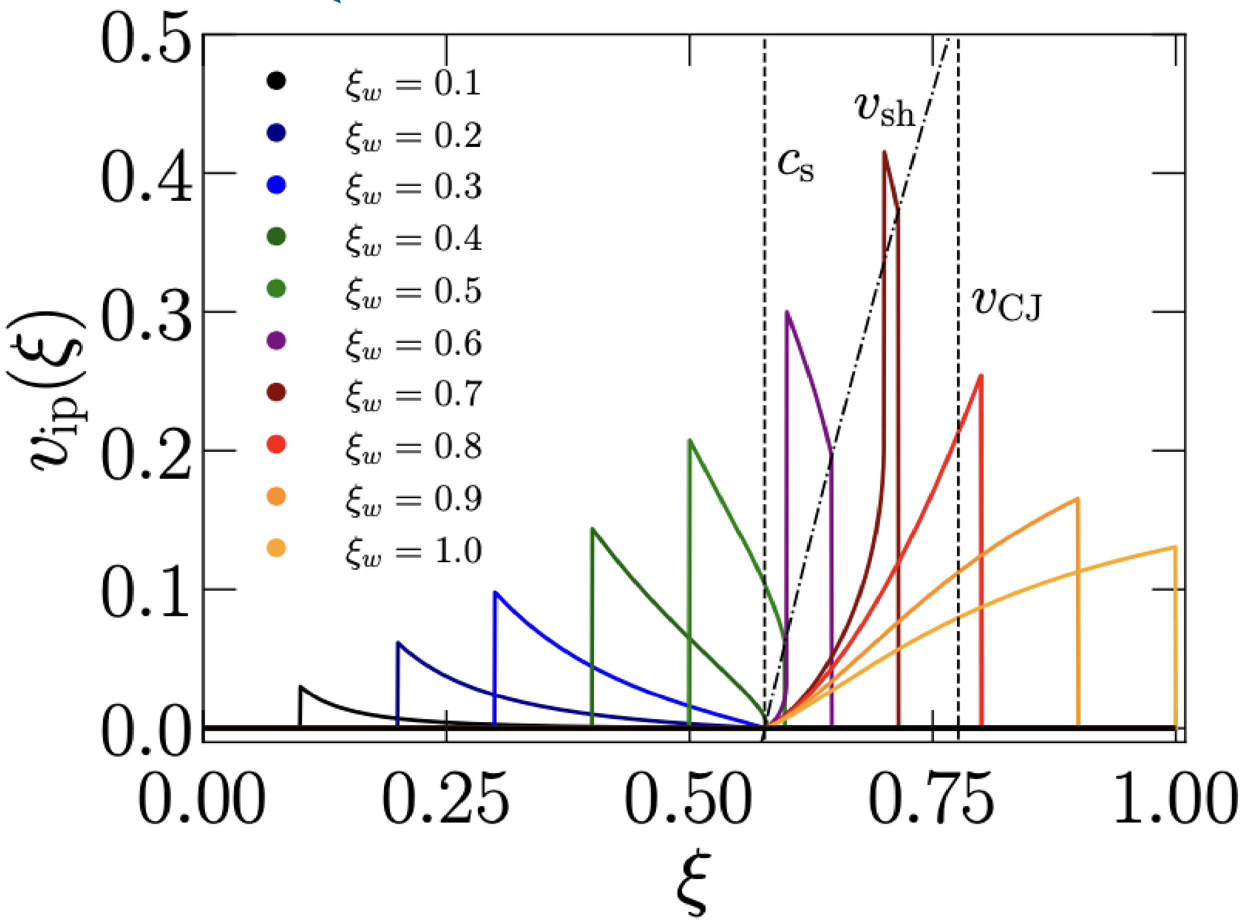
- Compact support
  $v_{ip}(\xi) \neq 0 \quad for \;\; \xi_b < \xi < \xi_f$

- Discontinuity at ξ_w

- Deflagrations and hybrids have an additional discontinuity at $\xi = v_{sh}$

pip install cosmoGW

$$v_{CJ}(\alpha) = \frac{1 + \sqrt{\alpha(2 + 3\alpha)}}{\sqrt{3}(1 + \alpha)}$$



Evolution of the fluid perturbations: *before* collisions

The kinetic spectrum in the bubble expansion phase is an average over stochastic realizations

$$\left\langle v_i(t,\boldsymbol{k}) v_j^*(t,\boldsymbol{k}') \right\rangle_{\boldsymbol{x}_0^{(n)},\, t_0^{(n)}}$$

$$\mathbf{v}^{(\mathrm{n})}(\mathrm{t},\mathbf{k}) = -\mathrm{i}\left[\mathrm{t}^{(\mathrm{n})}\right]^3 \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{x}_0^{(\mathrm{n})}}\,\hat{\mathbf{k}}\, f'(z)$$

$$f'(z) = -4\pi \int_0^\infty \mathrm{j}_1(\mathrm{z}\xi)\, \xi^2\, \mathrm{v}_{\mathrm{ip}}(\xi)\, \mathrm{d}\xi$$

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Average over nucleation locations (homogeneously distributed)

Properties of $|f'(z)|^2$

$$\left\langle v_i(t,\boldsymbol{k}) v_j^*(t,\boldsymbol{k}')\right\rangle_{\boldsymbol{x}_0^{(n)}} = \widehat{\boldsymbol{k}}_i\,\widehat{\boldsymbol{k}}_j\,\delta^{(3)}(\boldsymbol{k}-\boldsymbol{k}')\,n_b(t)(t-t_0)^6|f'(z)|^2$$

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From causality

Large scales $k = z/t^{(n)} \to 0$

$$|f'(z)|^2 \to |f_0'|^2\, z^2$$

Properties of $|f'(z)|^2$

$$\left\langle v_i(t,\boldsymbol{k}) v_j^*(t,\boldsymbol{k}')\right\rangle_{\boldsymbol{x}_0^{(n)}} = \hat{\boldsymbol{k}}_i\, \hat{\boldsymbol{k}}_j\, \delta^{(3)}(\boldsymbol{k}-\boldsymbol{k}')\, n_b(t)(t-t_0)^6 |f'(z)|^2$$

From causality

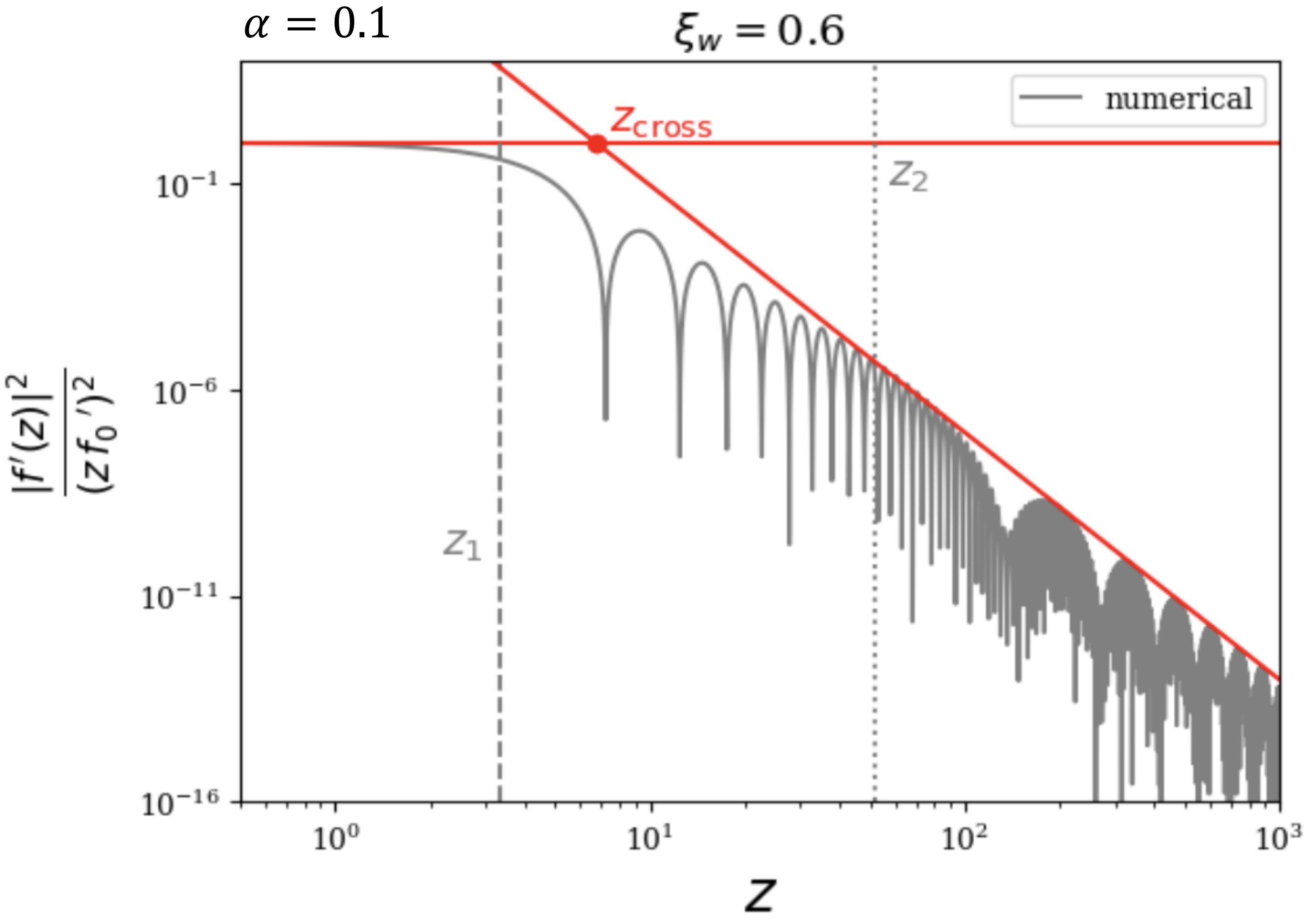
Large scales $k = z/t^{(n)} \to 0$ $\qquad |f'(z)|^2 \to |f_0'|^2\, z^2$

Small scales $k = z/t^{(n)} \to \infty$ $\qquad |f'(z)|^2 \to |f_\infty'|^2\, z^{-4}$

From the discontinuities of $v_{ip}(\xi)$

Properties of $|f'(z)|^2$

The $\sim z^2$ ends around $z_1 \approx \frac{3\pi}{2}\left(\xi_f + \xi_b\right)^{-1}$



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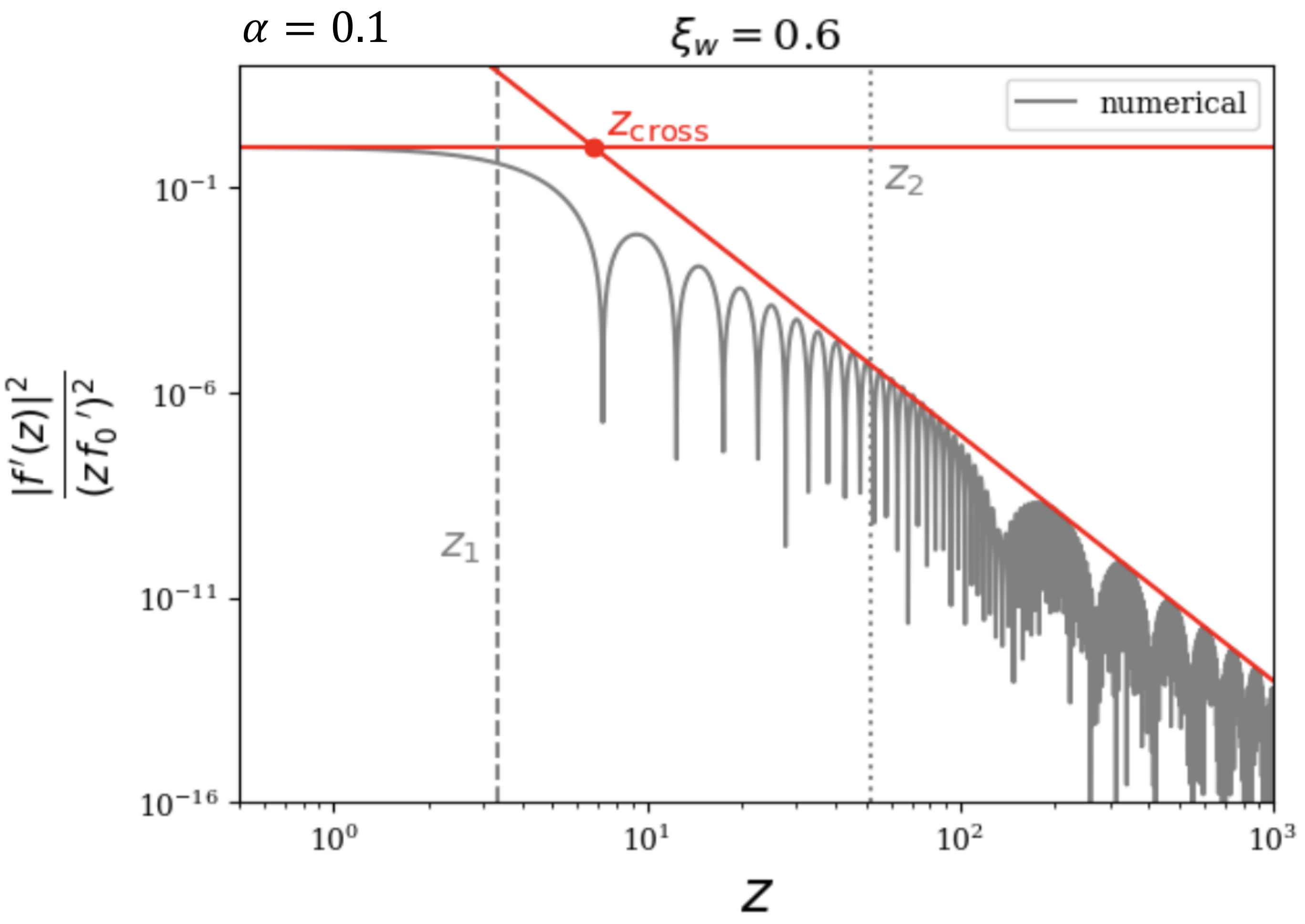

The $\sim z^{-4}$ begins around

$$z_2 \approx \pi \times \begin{cases} (\xi_f - \xi_b)^{-1} & (\xi_w < c_s) \\ (\xi_f - \xi_w)^{-1} & (c_s < \xi_w < v_{CJ}) \\ (\xi_f - \xi_b)^{-1} & (\xi_w > v_{CJ}) \end{cases}$$

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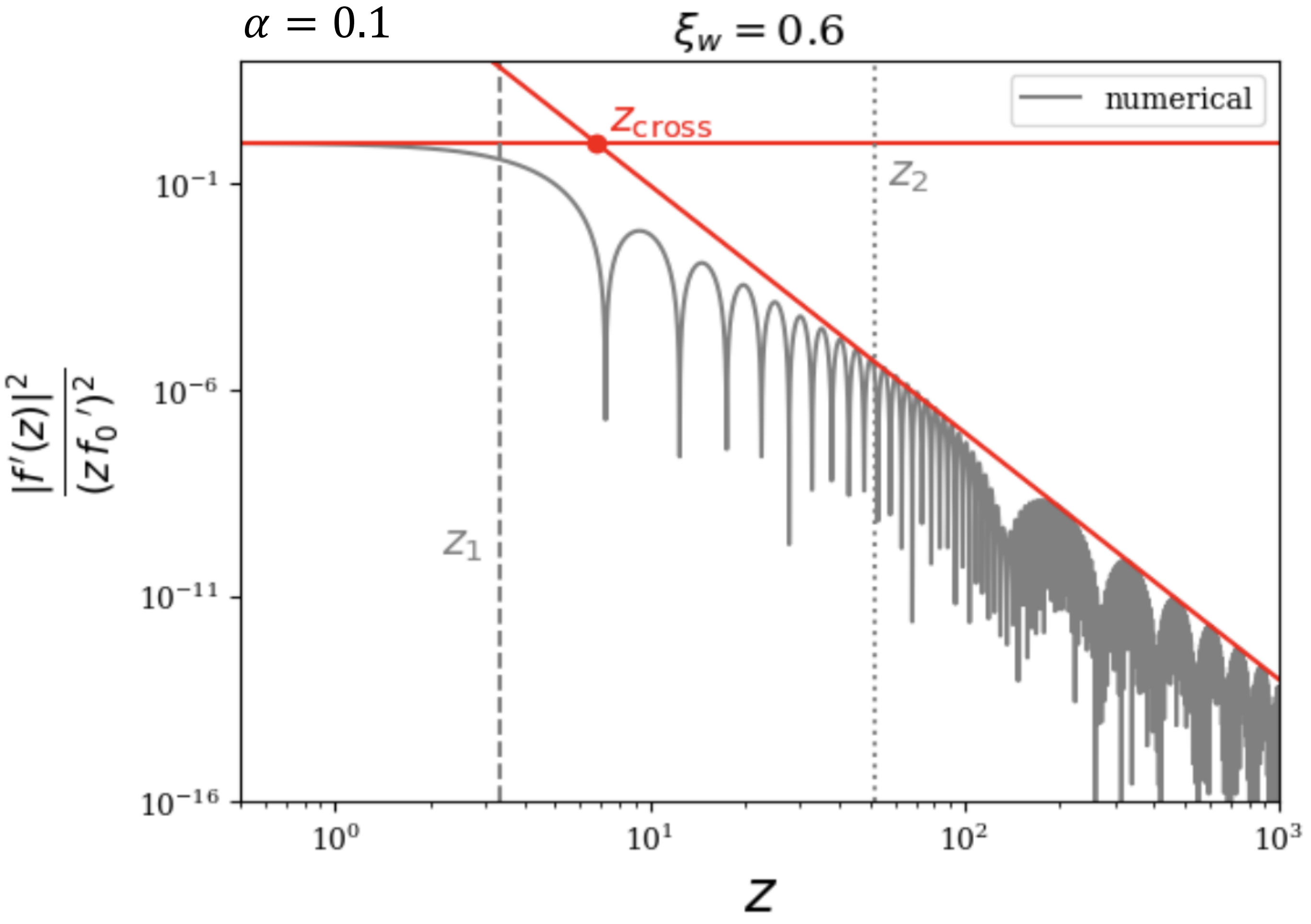

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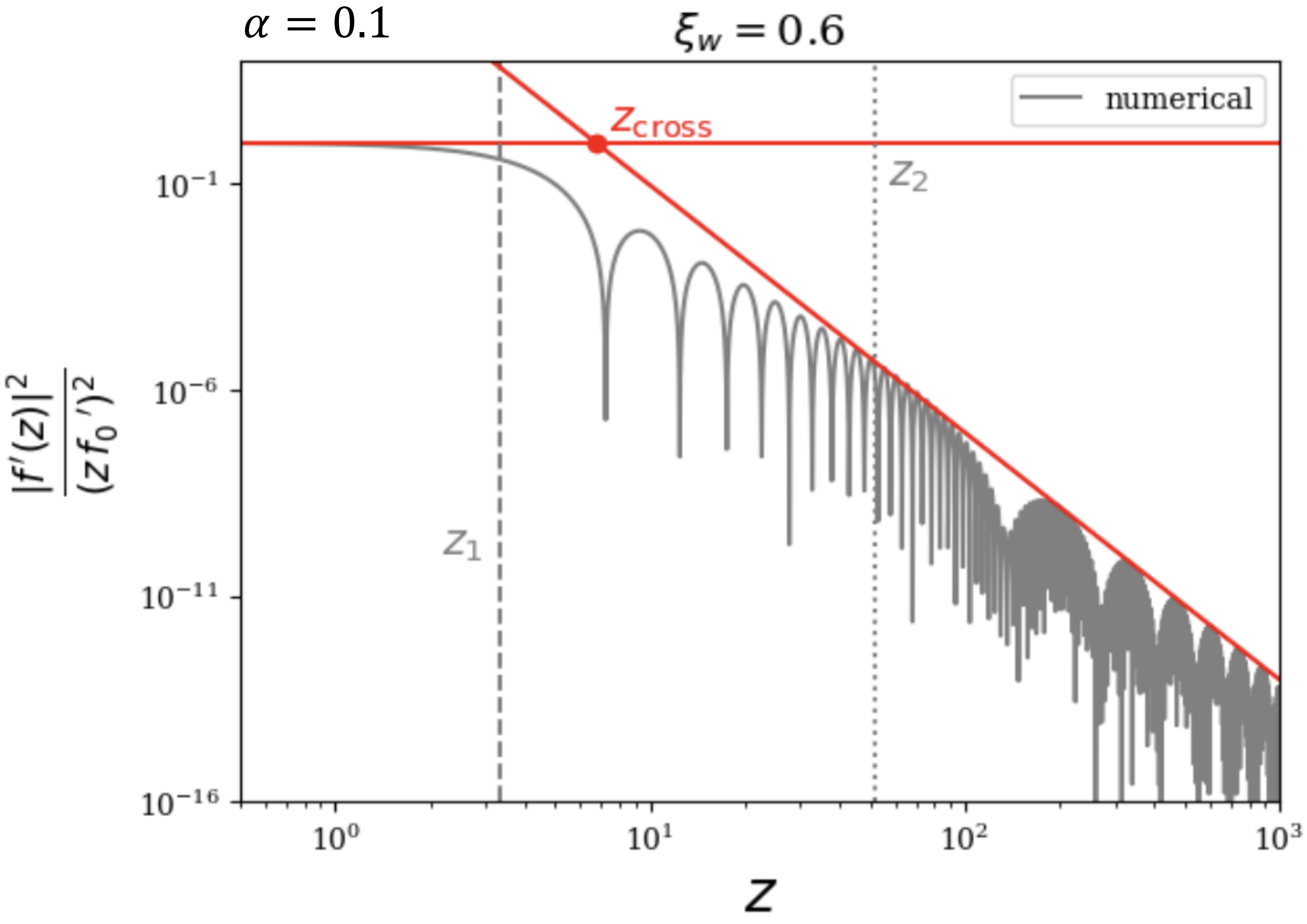
$\xi_f - \xi_b \propto \Delta R_*$ (sound shell thickness)

Properties of $|f'(z)|^2$

The $\sim z^2$ ends around $z_1 \approx \frac{3\pi}{2}(\xi_f + \xi_b)^{-1}$



The $\sim z^{-4}$ begins around

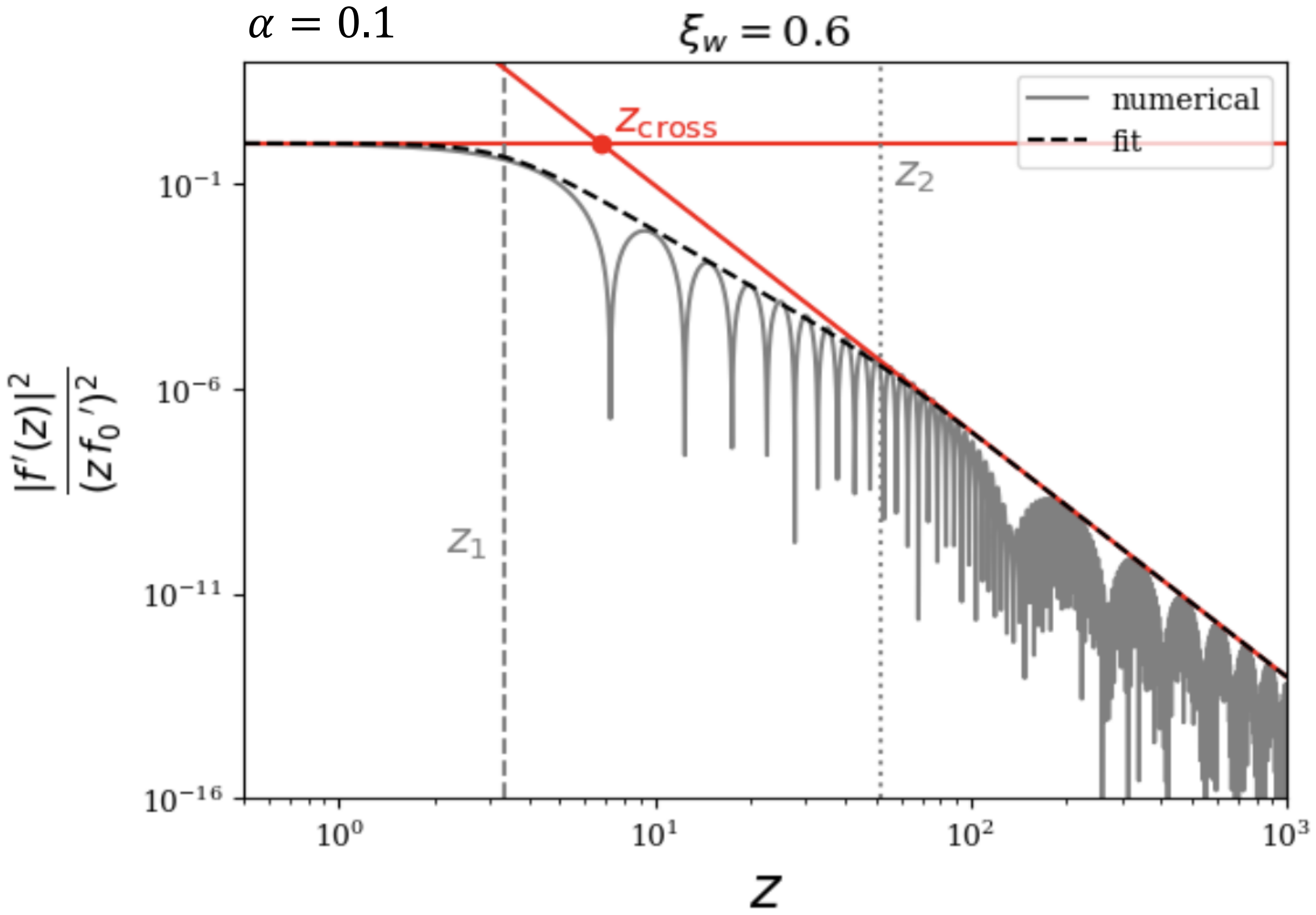
$$z_2 \approx \pi \times \begin{cases} (\xi_f - \xi_b)^{-1} & (\xi_w < c_s) \\ (\xi_f - \xi_w)^{-1} & (c_s < \xi_w < v_{CJ}) \\ (\xi_f - \xi_b)^{-1} & (\xi_w > v_{CJ}) \end{cases}$$

$\xi_f - \xi_b \propto \Delta R_*$ (sound shell thickness)

$\xi_f - \xi_w = \xi_{sh} - \xi_w$ distance between discontinuities (for hybrids)

Properties of $|f'(z)|^2$

$$|f'(z)|^2_{env} = |f_0'|^2 \, z^2 \left[1+\left(\frac{z}{z_1}\right)^{a_1}\right]^{\frac{\gamma-2}{a_1}} \left[1+\left(\frac{z}{z_2}\right)^{a_2}\right]^{\frac{-\gamma-4}{a_2}}$$



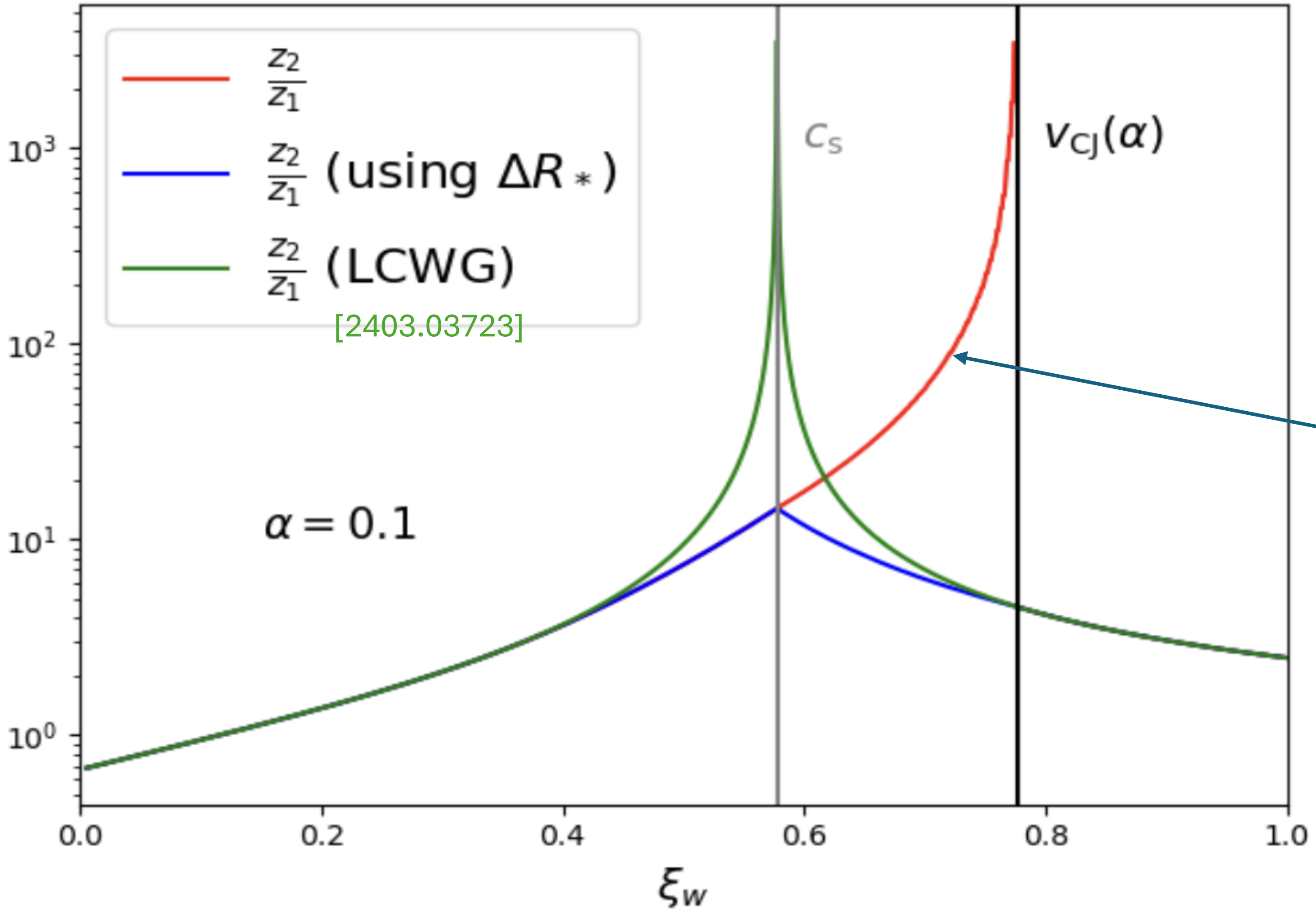
Double broken power law fit

$$\gamma = 2\left[1 - 3\frac{\log(z_2/z_{cross})}{\log(z_2/z_1)}\right]$$

Scales of $|f'(z)|^2$

$$z_1 \approx \frac{3\pi}{2}\left(\xi_f + \xi_b\right)^{-1}$$

$$z_2 \approx \pi \times \begin{cases} \left(\xi_f - \xi_b\right)^{-1} & (\xi_w < c_s) \\ \left(\xi_f - \xi_w\right)^{-1} & (c_s < \xi_w < v_{CJ}) \\ \left(\xi_f - \xi_b\right)^{-1} & (\xi_w > v_{CJ}) \end{cases}$$



Much broader spectrum for hybrids than using

$z_2 = \pi \times \left(\xi_f - \xi_b\right)^{-1} \propto \Delta R_*^{-1}$

$z_2 = \pi \times |c_s - \xi_w|^{-1}$ (Lisa Cosmology Working Group)

Evolution of the fluid perturbations: *before* collisions

The kinetic spectrum in the bubble expansion phase
is an average over stochastic realizations

$$\left\langle v_i(t,\boldsymbol{k}) v_j^*(t,\boldsymbol{k}') \right\rangle_{\boldsymbol{x}_0^{(n)},\, t_0^{(n)}}$$

← Average over nucleation times

Evolution of the fluid perturbations: *across* collisions

The kinetic spectrum in the bubble expansion phase
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$$\left\langle v_i(t,\boldsymbol{k}) v_j^*(t,\boldsymbol{k}') \right\rangle_{\boldsymbol{x}_0^{(n)},\, t_0^{(n)}}$$

← Average over nucleation times
and collision times

Evolution of the fluid perturbations: *across* collisions

The kinetic spectrum in the bubble expansion phase
is an average over stochastic realizations

$$\left\langle v_i(t,\boldsymbol{k})v_j^*(t,\boldsymbol{k}')\right\rangle_{\boldsymbol{x}_0^{(n)},\,t_0^{(n)}}$$

← Average over nucleation times
and collision times

We can model the nucleation history with a normalized lifetime distribution $\nu(T)$

$$F_L(t_{coll},k) = n_b(t_{coll})\int_0^\infty dT\,\nu(T)T^6|f'(kT)|^2$$

Kinetic spectrum at collisions

Hindmarsh & Hijazi [1909.10040]

Evolution of the fluid perturbations: *across* collisions

$$F_L(t_{coll}, k) = n_b(t_{coll}) \int_0^\infty dT\, \nu(T) T^6 |f'(kT)|^2$$ ← Kinetic spectrum at collisions

Large scales $k \to 0$ $F_L \to k^2 F_L^0$ k^2 ends around $k_1 \simeq \beta \frac{z_1}{5.7}$

(exponential nucleation)

Small scales $k \to \infty$ $F_L \to k^{-4} F_L^{env}$ k^{-4} starts around $k_2 \simeq \beta \frac{z_2}{2.4}$

Consequences for the gravitational wave spectrum

$$\Omega_{GW}(\tau_0, k) = 3\,\mathcal{T}_{GW} \iint_{\tau_*}^{\min[\tau_0, \tau_{fin}]} \frac{d\tau_1}{\tau_1}\frac{d\tau_2}{\tau_2} \cos k(\tau_0 - \tau_1) \cos k(\tau_0 - \tau_2)\, E_\Pi(k, \tau_1, \tau_2)$$

UETC for sound-waves computed from the kinetic spectrum

Hindmarsh & Hijazi [1909.10040]

Consequences for the gravitational wave spectrum

$$\Omega_{GW}(\tau_0, k) = 3\,\mathcal{T}_{GW} \iint_{\tau_*}^{\min[\tau_0, \tau_{fin}]} \frac{d\tau_1}{\tau_1}\frac{d\tau_2}{\tau_2} \cos k(\tau_0 - \tau_1) \cos k(\tau_0 - \tau_2)\, E_\Pi(k, \tau_1, \tau_2)$$

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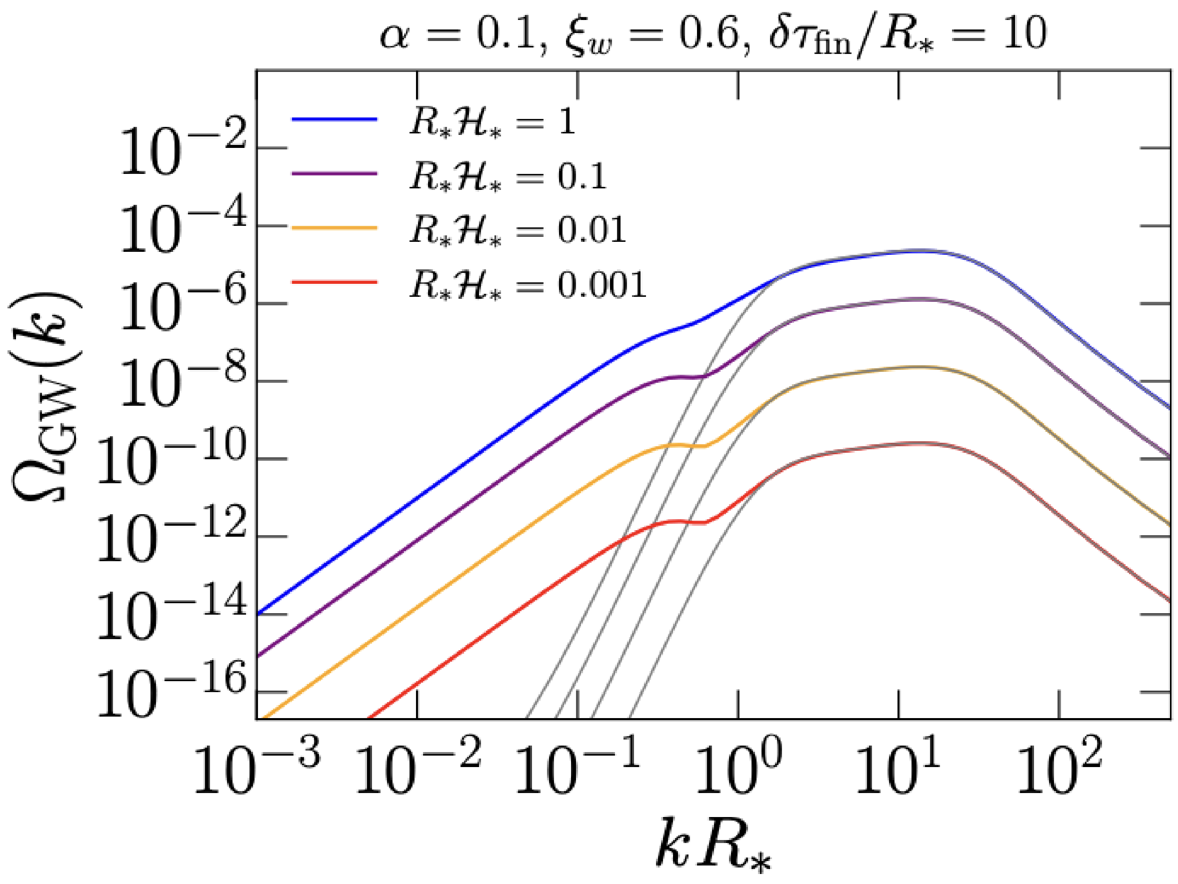


Double broken power law fit for the peak of Ω_{GW} with scales

$$k_1^{GW} \approx 1.2 \times k_1$$

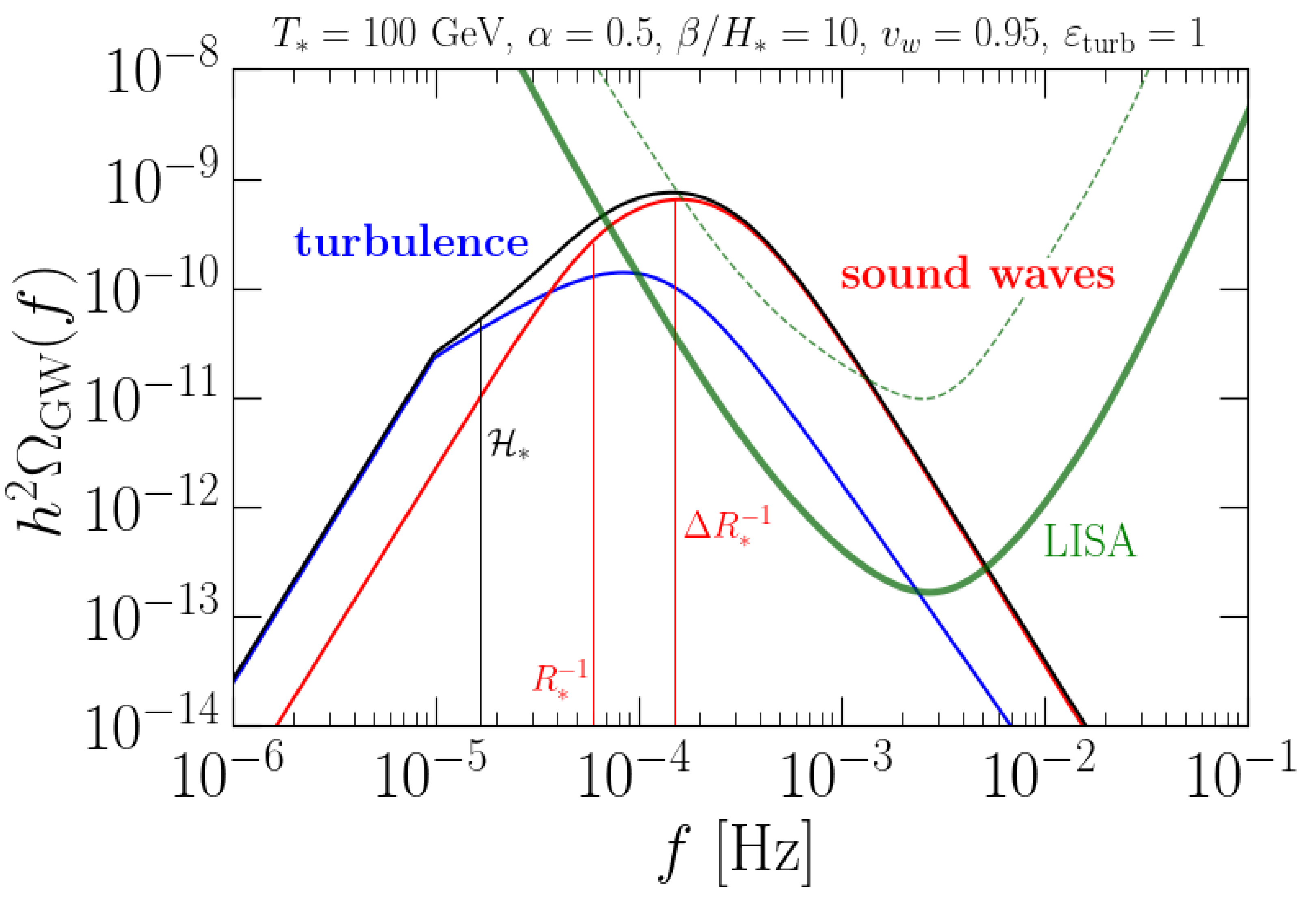
$$k_2^{GW} \approx 1.2 \times k_2$$

Roper Pol, Procacci, Caprini [2308.12943]

Gravitational Waves from decaying turbulence

[*Ongoing work* in collaboration with C. Caprini, A. Roper Pol, M. Salomé]

Introduction: first-order phase transitions and gravitational waves



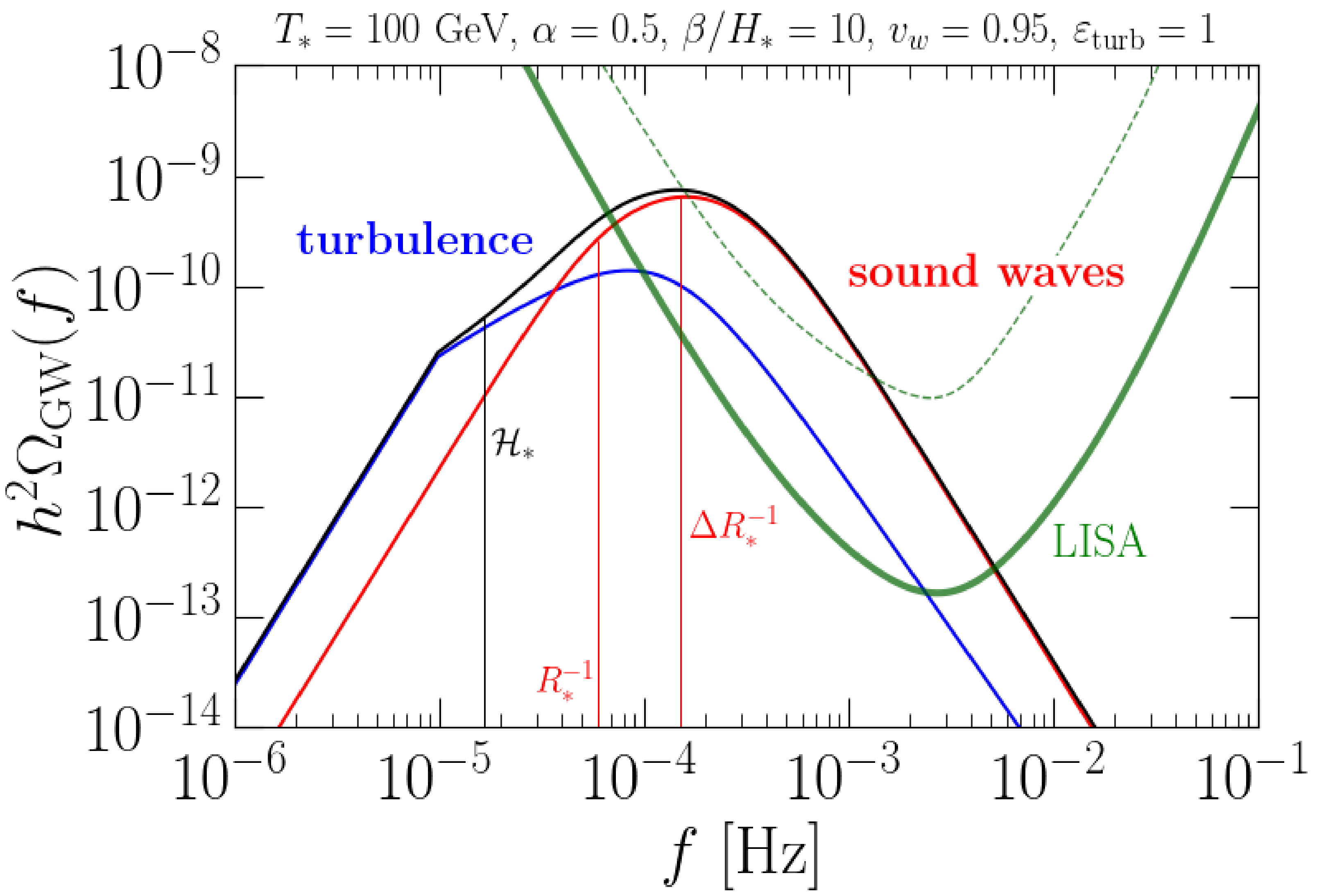
Constant-in-time model

Roper Pol, Caprini et al. [2201.05630]

← Credits: Alberto Roper Pol

Lisa Cosmology Working Group [2403.03723]

Introduction: first-order phase transitions and gravitational waves



Constant-in-time model

Roper Pol, Caprini et al. [2201.05630]

How long does it take for turbulence to develop?

Which fraction of energy goes into it?

How does the sourcing period affect the final GW spectrum?

How does turbulence evolve in the fully relativistic regime?

← Credits: Alberto Roper Pol

Lisa Cosmology Working Group [2403.03723]

Gravitational Waves from decaying MHD turbulence

- In First-Order Phase Transitions scalar field gradients can generate magnetic fields (Vachaspati et al. 2021) which can also be amplified by hydrodynamic turbulence, leading, due to the high conductivity of the plasma (Arnold et al. 2003), to MHD turbulence

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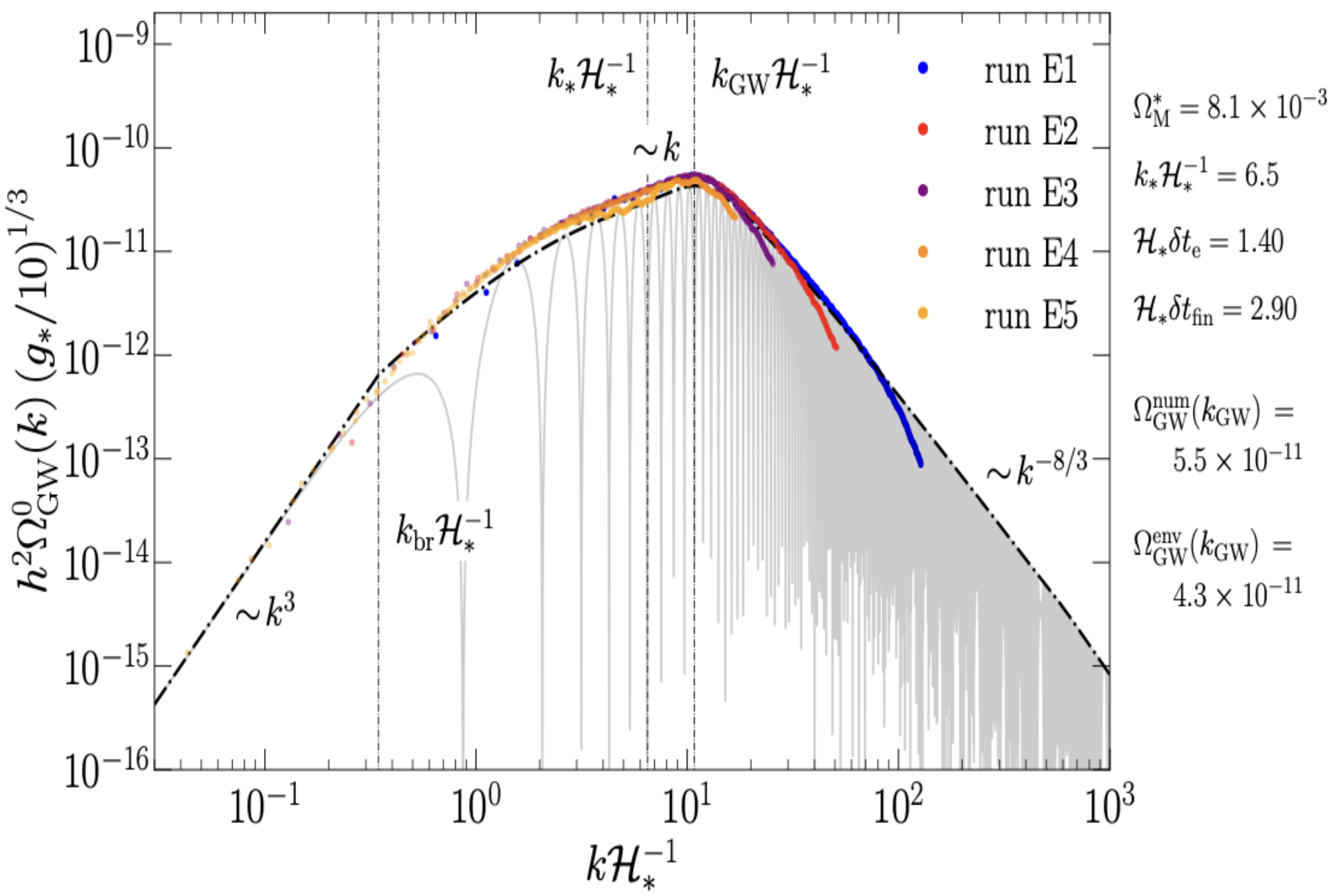
- The GW spectrum from numerical simulations of decaying MHD turbulence can be described with the constant-in-time model (Roper Pol et al. [2201.05630])



Constant-in-time model for the UETC of the source

$$\Omega_{GW}(\tau_0, k) = 3\, \mathcal{T}_{GW} \iint_{\tau_*}^{\min[\tau_0, \tau_{fin}]} \frac{d\tau_1}{\tau_1} \frac{d\tau_2}{\tau_2} \cos k(\tau_0 - \tau_1) \cos k(\tau_0 - \tau_2)\, E_\Pi(k, \tau_1, \tau_2)$$

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Assuming that the source is slowly decaying* for $\tau_* < \tau < \tau_{fin}$ $\longrightarrow$ $E_\Pi(k, \tau_1, \tau_2) = E_\Pi^*(k)$

*with respect to the light crossing time at wavenumber k

Constant-in-time model for the UETC of the source

$$\Omega_{GW}(\tau_0, k) = 3\,\mathcal{T}_{GW} \iint_{\tau_*}^{\min[\tau_0,\tau_{fin}]} \frac{d\tau_1}{\tau_1}\frac{d\tau_2}{\tau_2} \cos k(\tau_0-\tau_1)\cos k(\tau_0-\tau_2)\, E_\Pi(k,\tau_1,\tau_2)$$

Assuming that the source is slowly decaying* for $\tau_* < \tau < \tau_{fin}$ ⟶ $E_\Pi(k,\tau_1,\tau_2) = E_\Pi^*(k)$

$$\Omega_{GW}(\tau_0, k) = 3\,\mathcal{T}_{GW} E_\Pi^*(k) \int_{\tau_*}^{\min[\tau_0,\tau_{fin}]} \int_{\tau_*}^{\min[\tau_0,\tau_{fin}]} \frac{d\tau_1}{\tau_1}\frac{d\tau_2}{\tau_2} \cos k(\tau_0-\tau_1)\cos k(\tau_0-\tau_2)$$

$$\equiv 3\,\mathcal{T}_{GW} E_\Pi^*(k)\, \Delta^2(k,\tau_0)$$

*with respect to the light crossing time at wavenumber k

Constant-in-time model for the UETC of the source

$$\Omega_{GW}(\tau_0,k) = 3\,\mathcal{T}_{GW} \iint_{\tau_*}^{\min[\tau_0,\tau_{fin}]} \frac{d\tau_1}{\tau_1}\frac{d\tau_2}{\tau_2} \cos k(\tau_0-\tau_1)\cos k(\tau_0-\tau_2)\, E_\Pi(k,\tau_1,\tau_2)$$

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$$\equiv 3\,\mathcal{T}_{GW} E_\Pi^*(k)\, \Delta^2(k,\tau_0)$$

$$\mathcal{T}_{GW} = \left(\frac{a_*}{a_0}\right)^4 \left(\frac{H_*}{H_0}\right)^2 \approx 1.6 \times 10^{-5} \left(\frac{g_*}{100}\right)^{-\frac{1}{3}}$$

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$$\Omega_{GW}(\tau_0,k) = 3\,\mathcal{T}_{GW} \iint_{\tau_*}^{\min[\tau_0,\tau_{fin}]} \frac{d\tau_1}{\tau_1}\frac{d\tau_2}{\tau_2} \cos k(\tau_0-\tau_1)\cos k(\tau_0-\tau_2)\, E_\Pi(k,\tau_1,\tau_2)$$

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$$E_\Pi^*(k) \propto \langle \Pi_{ij}(\tau_*,k)\Pi_{ij}^*(\tau_*,k)\rangle$$

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Constant-in-time model for the UETC of the source

$$\Omega_{GW}(\tau_0,k) = 3\,\mathcal{T}_{GW} \iint_{\tau_*}^{\min[\tau_0,\tau_{fin}]} \frac{d\tau_1}{\tau_1}\frac{d\tau_2}{\tau_2} \cos k(\tau_0-\tau_1)\cos k(\tau_0-\tau_2)\, E_\Pi(k,\tau_1,\tau_2)$$

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$$\equiv 3\,\mathcal{T}_{GW} E_\Pi^*(k)\,\Delta^2(k,\tau_0)$$

$$\Delta(k,\tau_0) \equiv \int_{\tau_*}^{\min[\tau_0,\tau_{fin}]} \frac{d\tilde\tau}{\tilde\tau}\cos k(\tau_0-\tilde\tau)$$

$$\mathcal{T}_{GW} = \left(\frac{a_*}{a_0}\right)^4 \left(\frac{H_*}{H_0}\right)^2 \approx 1.6\times 10^{-5}\left(\frac{g_*}{100}\right)^{-\frac{1}{3}}$$

$$E_\Pi^*(k) \propto \langle \Pi_{ij}(\tau_*,k)\Pi_{ij}^*(\tau_*,k)\rangle$$

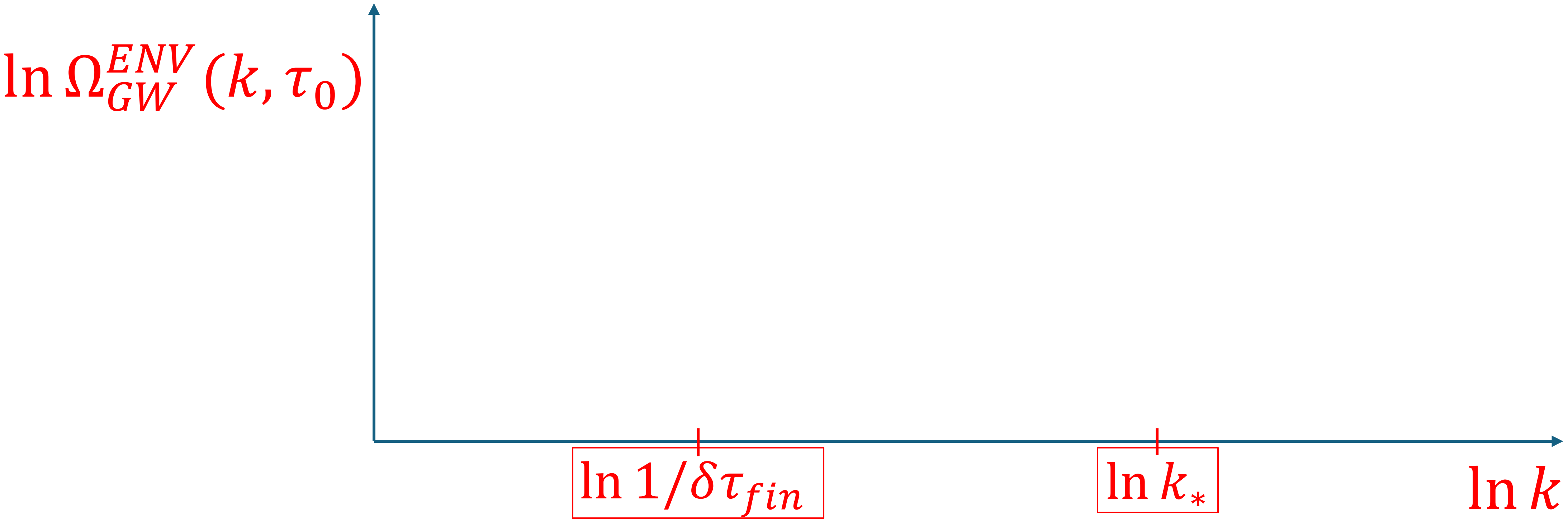
*with respect to the light crossing time at wavenumber k

Constant-in-time model for the UETC of the source

$$\Omega_{GW}(k,\tau_0) \equiv 3\, \mathcal{T}_{GW} E^*_\Pi(k)\, \Delta_0^2(k,\tau_{fin})$$

causality

Assuming for the UETC $E^*_\Pi(k) \sim \begin{cases} k^3 & k < k_* \\ k^{-b} & k > k_* \end{cases}$



Constant-in-time model for the UETC of the source

$$\Omega_{GW}(k,\tau_0) \equiv 3\,\mathcal{T}_{GW}\,E^*_\Pi(k)\,\Delta_0^2(k,\tau_{fin})$$

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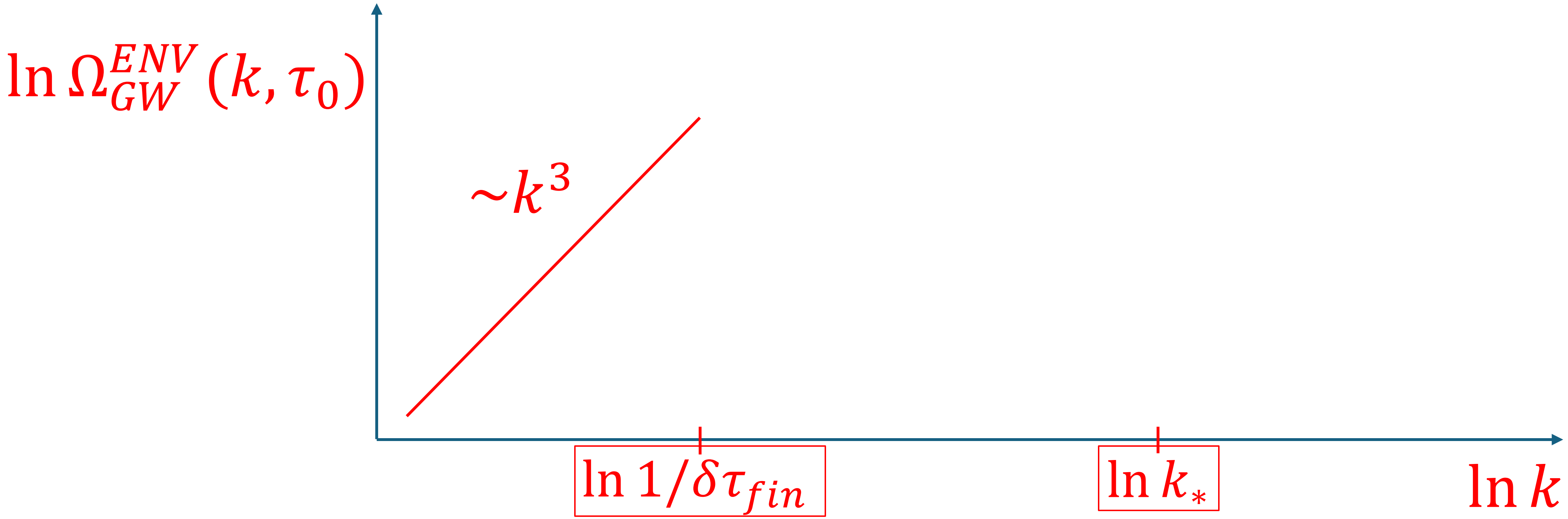

Constant-in-time model for the UETC of the source

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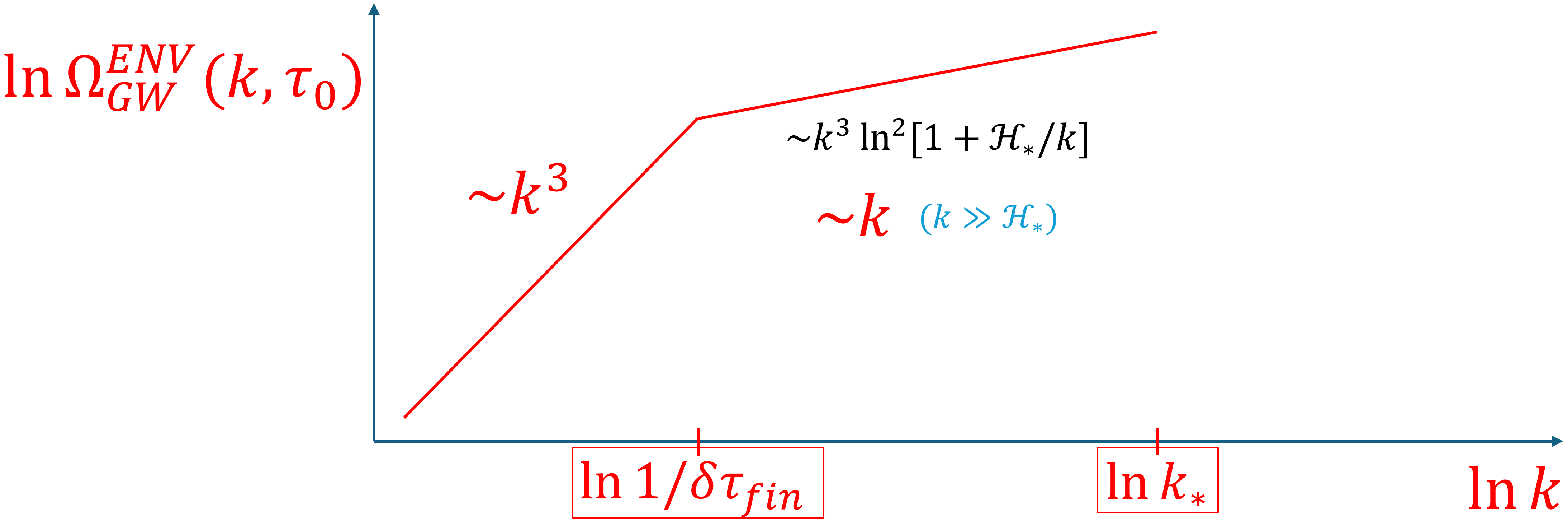
causality → k^3



Constant-in-time model for the UETC of the source

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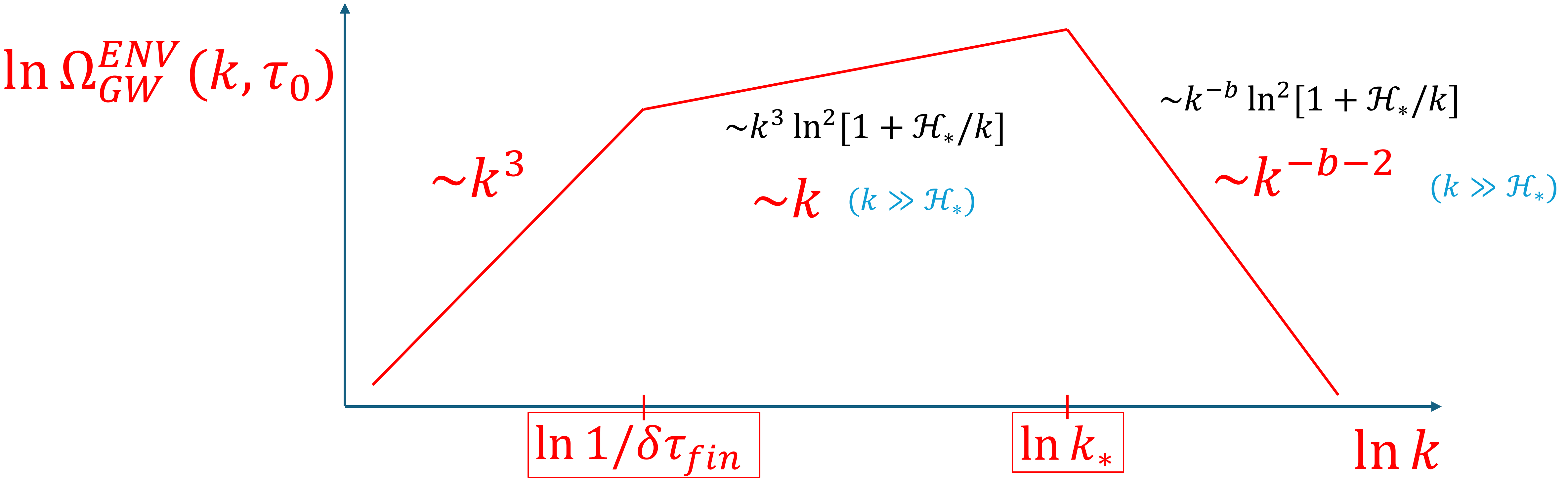
Assuming for the UETC $E_\Pi^*(k) \sim \begin{cases} k^3 & k < k_* \\ k^{-b} & k > k_* \end{cases}$ (causality: k^3)



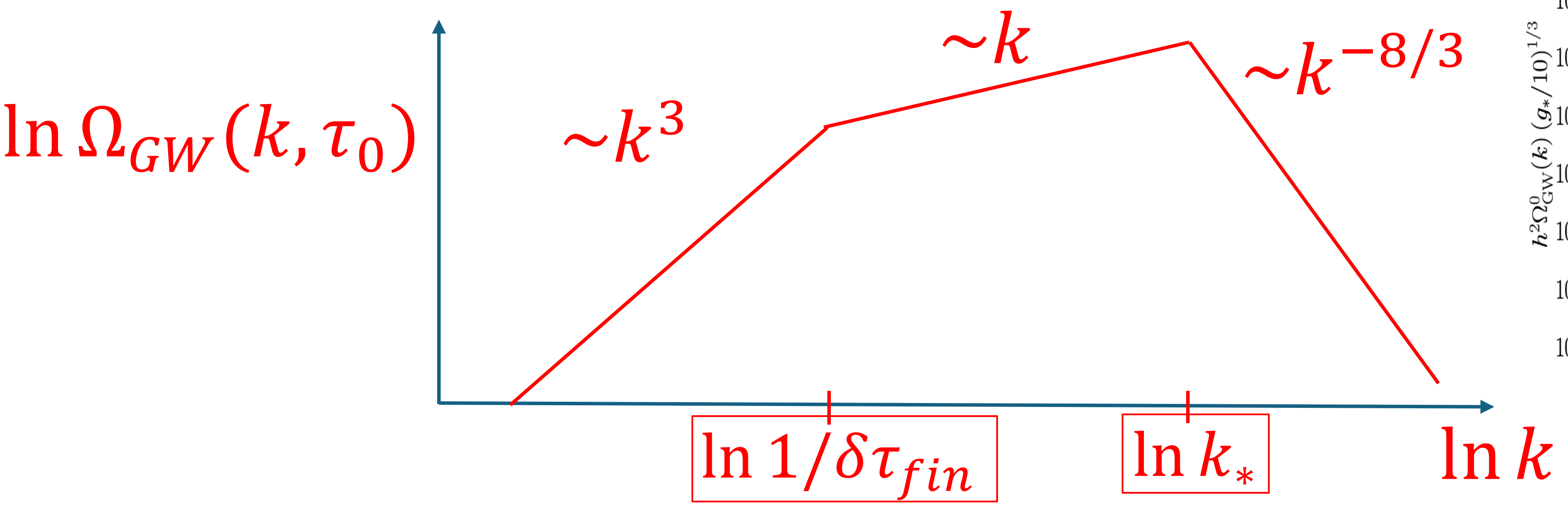
Gravitational Waves from decaying turbulence

For a purely vortical velocity field with a Von Kármán spectrum

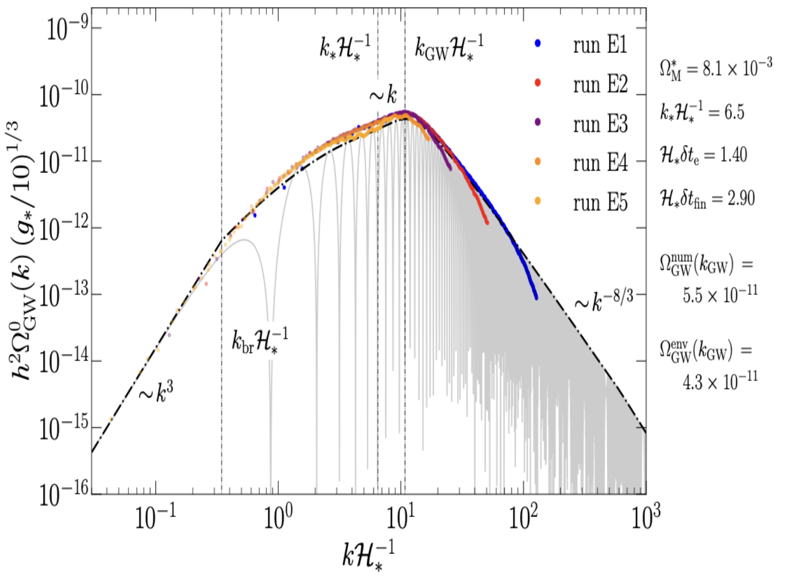
$$E_N^v(k) \sim \begin{cases} k^5 & (k/k_{peak} \to 0) \quad \textit{Batchelor} \\ k^{-2/3} & (k/k_{peak} \to \infty) \quad \textit{Kolmogorov} \end{cases}$$

$$E_\Pi(k) \sim \begin{cases} k^3 & (k/k_* \to 0) \\ k^{-2/3} & (k/k_* \to \infty) \end{cases}$$

GW spectrum envelope for vortical turbulence in the constant-in-time model (flat spacetime)



Roper Pol et al. [2201.05630]



Conclusions of part I

GW spectrum from sound waves (in the sound shell model) can be understood from the properties of the self-similar profiles and of the bubble nucleation history

For hybrids the GW peak scale is related to the distance between discontinuities instead of the sound-shell thickness (broader spectrum around the peak)

The contribution from slowly decaying MHD turbulence can be described with a constant-in-time UETC of the anisotropic stresses of the source

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How well do these models describe the results from numerical simulations?

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For turbulence also the generation phase (not only the decaying part) can be relevant in the final GW spectrum

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General case requires numerical simulations → See Part II

THANKS FOR YOUR ATTENTION!