14:00 → 15:30 **Post-processing and visualization** 1h 30m 4/3-006 - TH Conference Room

Speakers: Axel Brandenburg (Nordita), Philippe Bourdin (University of Graz), Simon Candelaresi (University of Augsburg)

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Postprocessing:

Prepare as much as possible during run time
Reading full snapshots in serial is slow

-> volume rendering
-> precompute voxels

Runtime analysis/output

  time series (5 steps to implement something new)
  slices
  spectra
  PDFs
  averages (xyaver, etc)
  sound file
```

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----------------------------------------
pc_read_.....
ls -l ~/pencil-code/idl/read/pc_read*

pc_read_var.pro
pc_read_ode.pro

pc_read_ts.pro

pc_read_xyaver.pro
pc_read_yzaver.pro
pc_read_xzaver.pro
pc_read_yaver.pro
pc_read_zaver.pro
pc_read_1d_aver.pro
pc_read_2d_aver.pro
pc_read_phiavg.pro
pc_read_phizaver.pro

pc_read_param.pro

pc_read_pdim.pro
pc_read_psize.pro
pc_read_pstalk.pro
pc_read_pvar.pro
pc_read_qdim.pro
pc_read_qvar.pro

pc_read_saffman.pro
modules/powerspectra/power.pro

pc_read_slice.pro
pc_read_video.pro
pc_read_videoslices.pro
```

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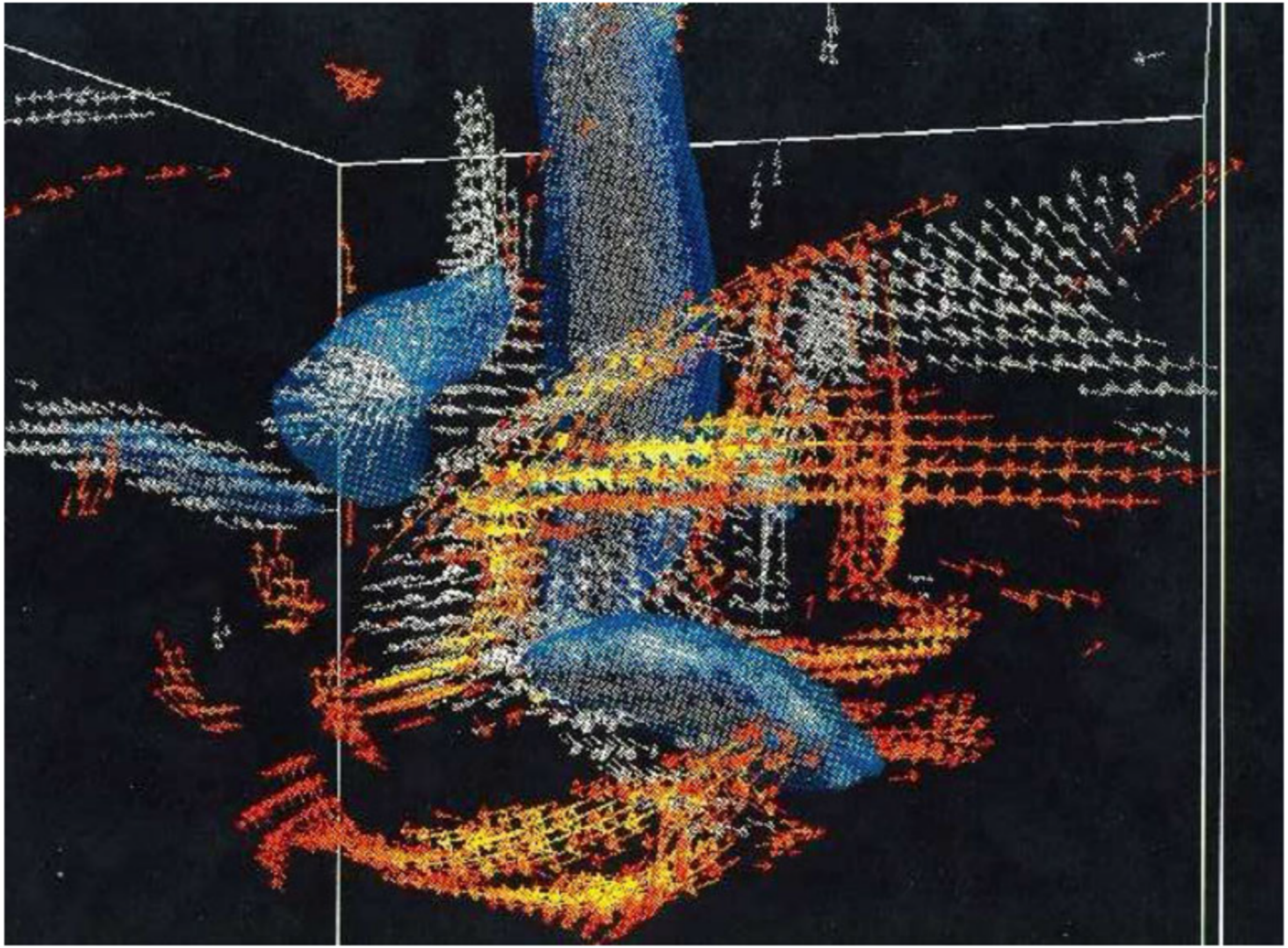
examples:

slices:
dardel:
scr/public_html/teach/PencilCode/EarlyUnivSchool/session1_run/const-nu-32768-ampl10-nu01
scr/public_html/teach/PencilCode/EarlyUnivSchool/session1_run/const-nu-32768-ampl10-nu02

/home/brandenb/data/isak/rel/3d/MGWp1024b_vw08_alphap5_L20_noexp_nu2em3/PNG_u2
vlc MGWp1024b_vw08_alphap5_L20_noexp_nu2em3_u2.mp4

spectra:
/home/brandenb/data/sayan/GW/P1024_k1_kf10c_rho_nonuni
cat data/varname.dat

GW accuracy
```



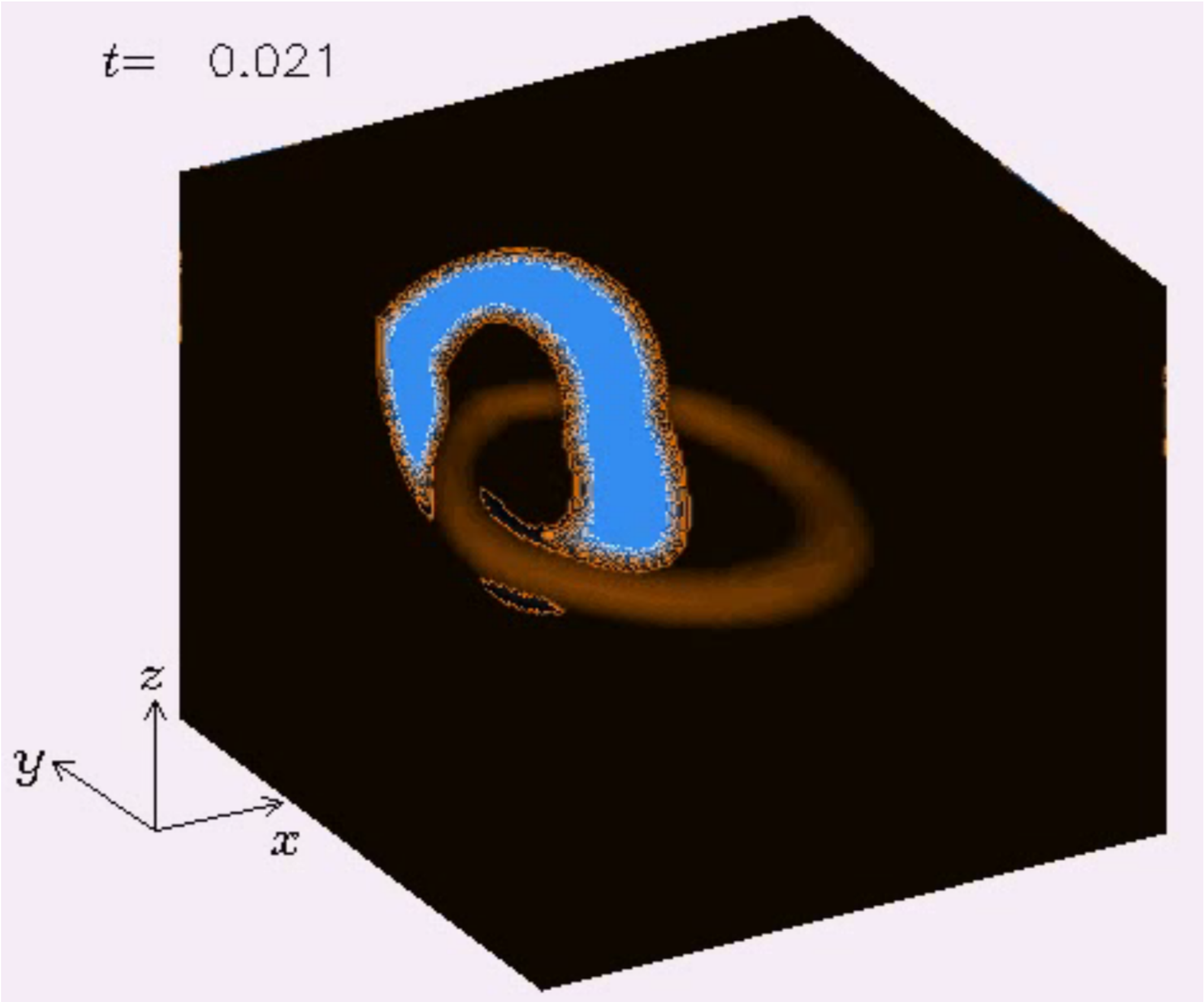
Vectors
above
threshold

3 Bins

Mutual obscuration as with lightsaber

$$\hat{\mathbf{n}} \cdot \nabla I = -\rho\kappa \left(I - S \right)$$

$$\rho\kappa \propto B^2, \quad S \propto B^2$$



Interlinked

$\vec{B}$ tube and

$\vec{\omega}$ tube.

$\Rightarrow$ Cross helicity

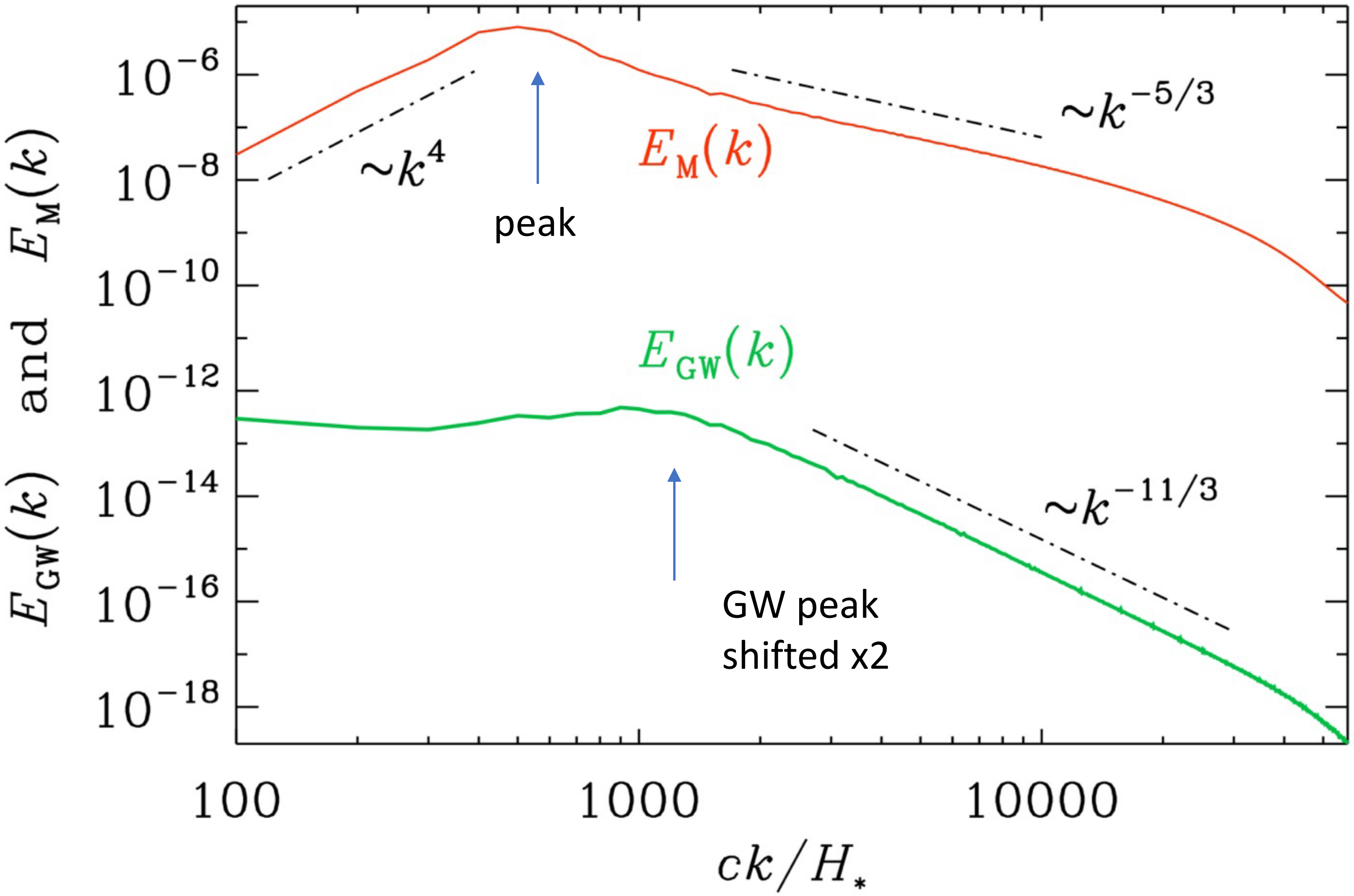
$$\int B \cdot (\nabla \times)^{-1} \omega \, dV$$

$$= \int B \cdot u \, dV$$

$$\neq 0 \quad \text{(conserved)}$$

Gravitational Waves (GWs)

Correspondence with (magnetohydrodynamic) turbulence



Roper Pol et al. (2020)

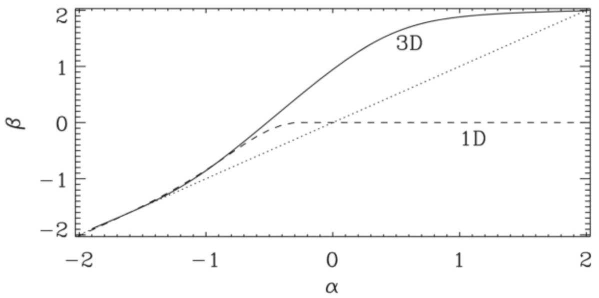
- Spectral energy per linear wavenumber interval
- $\Omega_{GW}(\ln k)=kE_{GW}$
- Forward cascade $k^{-5/3}$

$$\left(\partial_t^2 + 3H\partial_t - c^2\nabla^2\right) h_{ij}(\boldsymbol{x},t) = \frac{16\pi G}{c^2} T_{ij}^{\mathrm{TT}}(\boldsymbol{x},t)$$

- Relation between spectra:

$$\mathrm{Sp}(\dot{\mathbf{h}}) \approx k^2 \mathrm{Sp}(\mathbf{h}) \approx k^{-2} \mathrm{Sp}(\mathbf{T}),$$

GW slope by k^2 steeper
Peak at twice magnetic peak

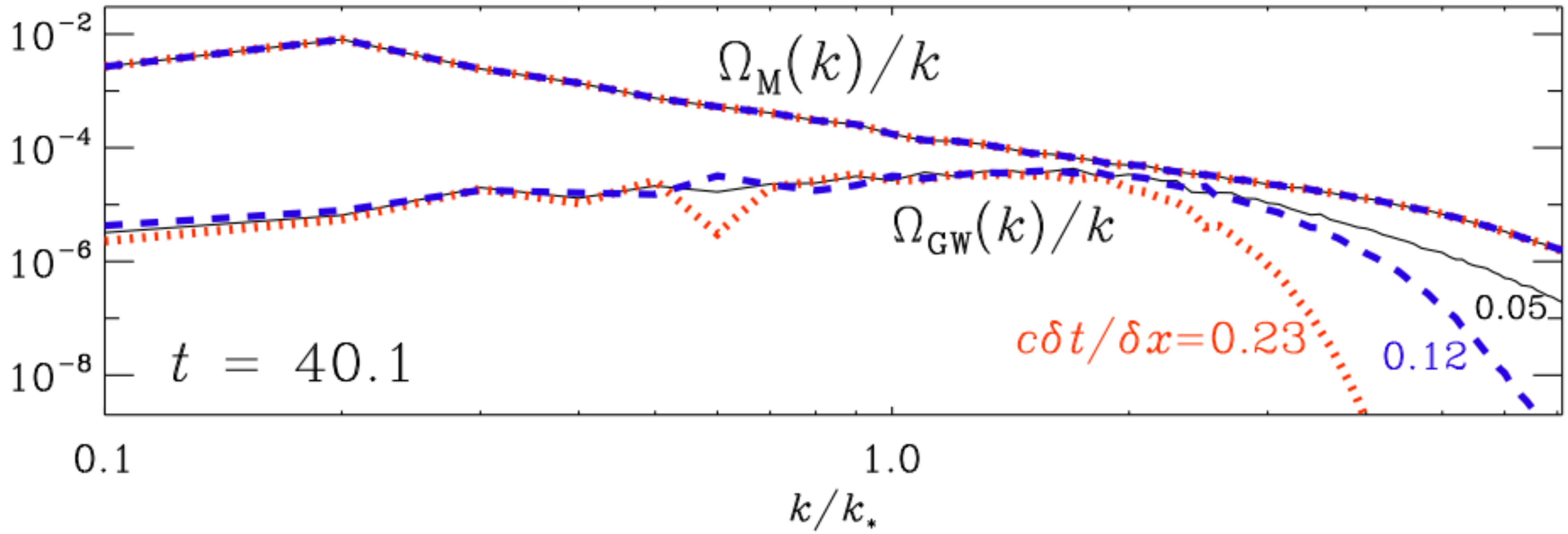


Inaccuracy of "usual" 3rd order Runge-Kutta

$$\begin{pmatrix} h_{ij} \\ h'_{ij} \end{pmatrix}_{t+\delta t} \equiv \boldsymbol{q}_i, \quad \text{where} \quad \boldsymbol{q}_i = \boldsymbol{q}_{i-1} + \beta_i \boldsymbol{w}_i, \quad \boldsymbol{w}_i = \alpha_i \boldsymbol{w}_{i-1} + \delta t \boldsymbol{Q}_{i-1}, \quad \text{(approach I)}.$$

with $\alpha_1 = 0$, $\alpha_2 = -5/9$, $\alpha_3 = -153/128$, $\beta_1 = 1/3$, $\beta_2 = 15/16$, $\beta_3 = 8/15$, and

$$\boldsymbol{q}_{i-1} \equiv \begin{pmatrix} h_{ij} \\ h'_{ij} \end{pmatrix}_t, \quad \boldsymbol{Q}_{i-1} \equiv \begin{pmatrix} h'_{ij} \\ c^2 \nabla^2 h_{ij} + \mathcal{G} T_{ij} \end{pmatrix}_t.$$



Alternative: exact solution for constant source between time steps

Consider: $$\ddot{h} + k^2 h = S$$

Solve as 2 first-order eqs

$$\begin{aligned} \dot{h} &= g \\ \ddot{h} \equiv \dot{g} &= -k^2 h + S \end{aligned}$$

General solution:

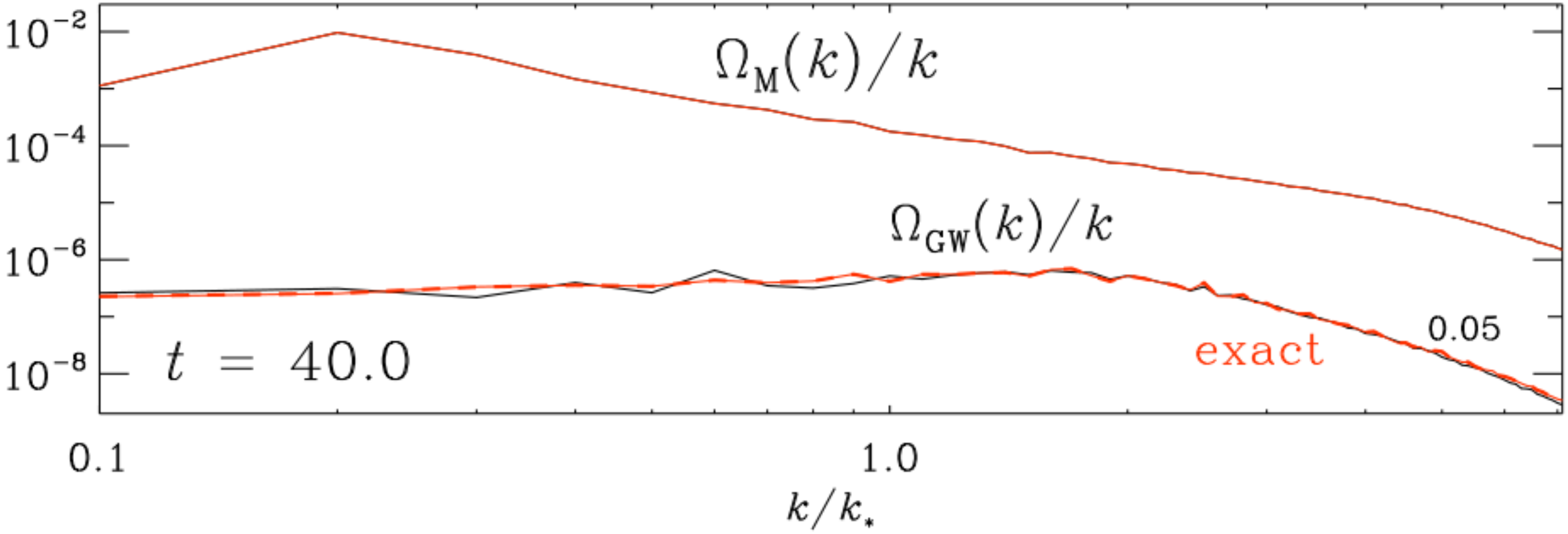
(h, g) at $t = 0$

$$\begin{aligned} h &= +A\cos kt + B\sin kt + k^{-2}S \\ g &= -Ak\sin kt + Bk\cos kt, \end{aligned}$$

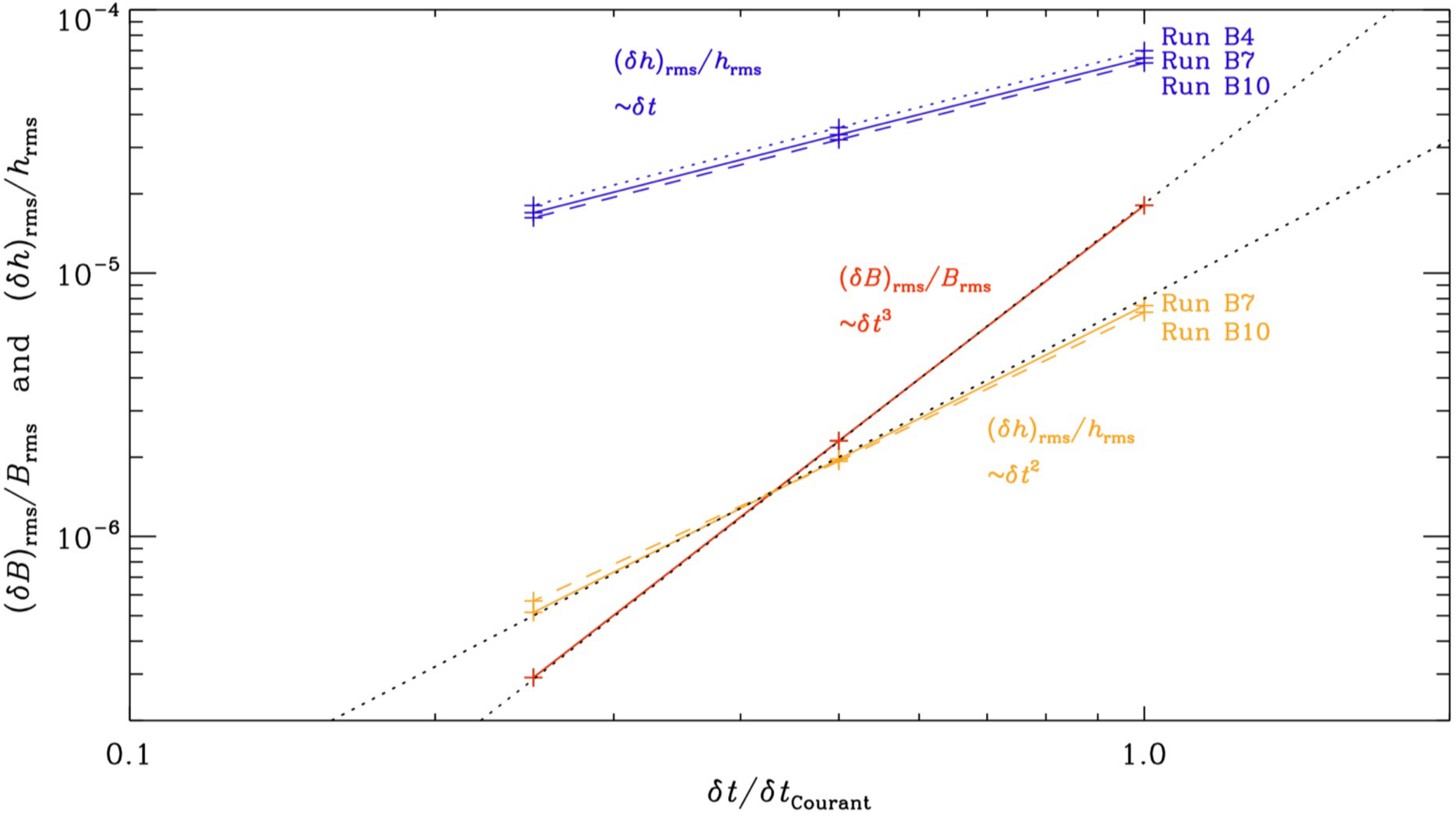
$$\begin{aligned} A &= h - k^{-2}S \\ B &= k^{-1}g \end{aligned}$$

$$\begin{aligned} h(\delta t) &= +(h - k^{-2}S)\cos k\delta t + k^{-1}g\sin k\delta t + k^{-2}S \\ g(\delta t) &= -(h - k^{-2}S)k\sin k\delta t + k^{-1}gk\cos k\delta t, \end{aligned}$$

$$\begin{pmatrix} kh - k^{-1}S \\ g \end{pmatrix}_{\rm new} = \begin{pmatrix} \cos k\delta t & \sin k\delta t \\ -\sin k\delta t & \cos k\delta t \end{pmatrix} \begin{pmatrix} kh - k^{-1}S \\ g \end{pmatrix}_{\rm current}$$



Dependence of accuracy on time step: only 1st order



Allowing linear variations between time steps

Taylor expand:

$$\begin{aligned} h &= +A\cos kt + B\sin kt + k^{-2}(S_0 + \dot{S}_0\delta t) \\ g &= -Ak\sin kt + Bk\cos kt + k^{-2}\,\dot{S}_0 \end{aligned}$$

Modified update involving δS

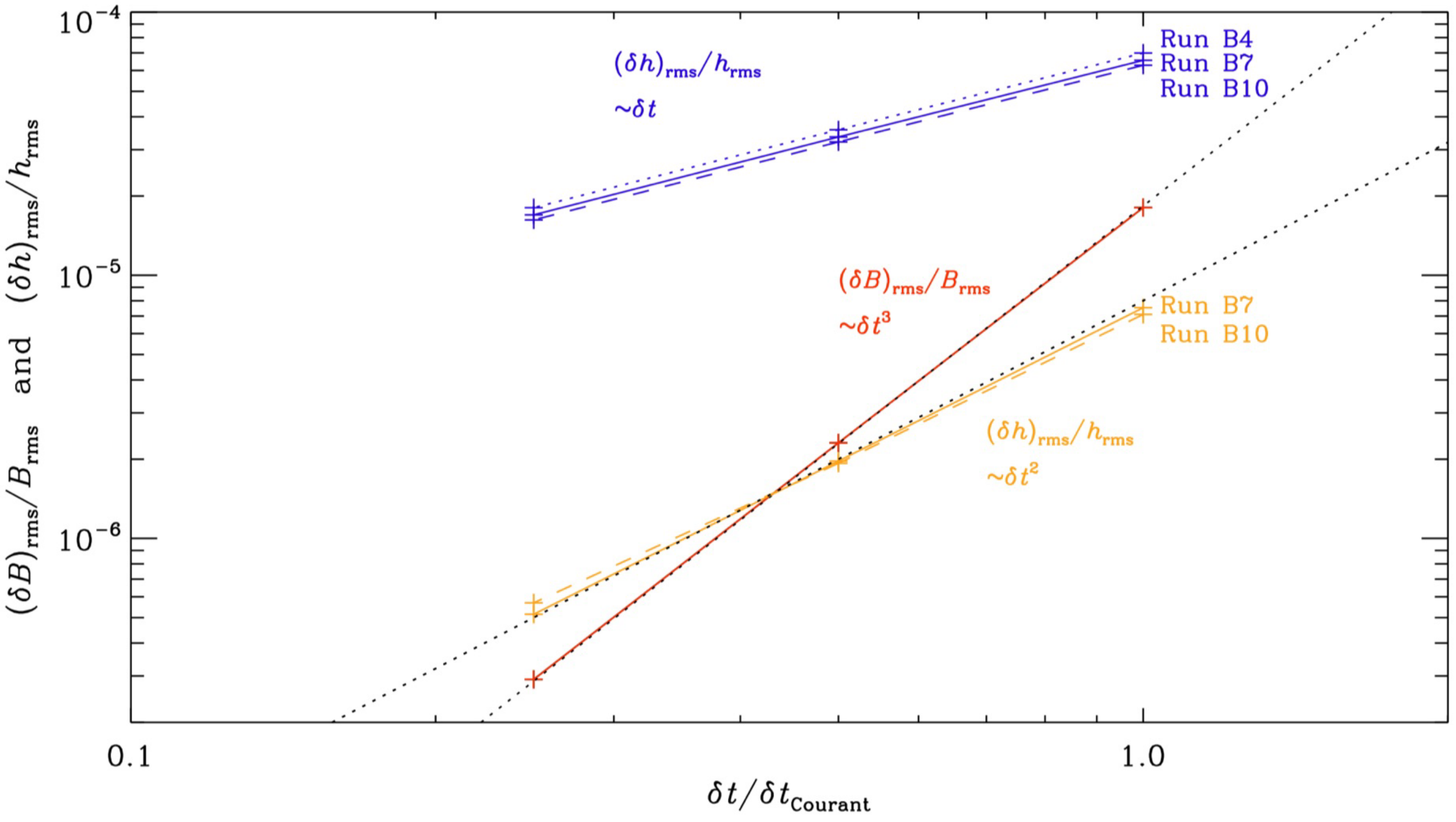
$$\begin{pmatrix} kh - k^{-1}(S_0 + \delta S) \\ g - k^{-2}\delta S/\delta t \end{pmatrix}_{\text{new}} = \begin{pmatrix} \cos k\delta t & \sin k\delta t \\ -\sin k\delta t & \cos k\delta t \end{pmatrix} \begin{pmatrix} kh - k^{-1}S \\ g - k^{-2}\delta S/\delta t \end{pmatrix}_{\text{current}}$$

Additional update to make it 2nd order:

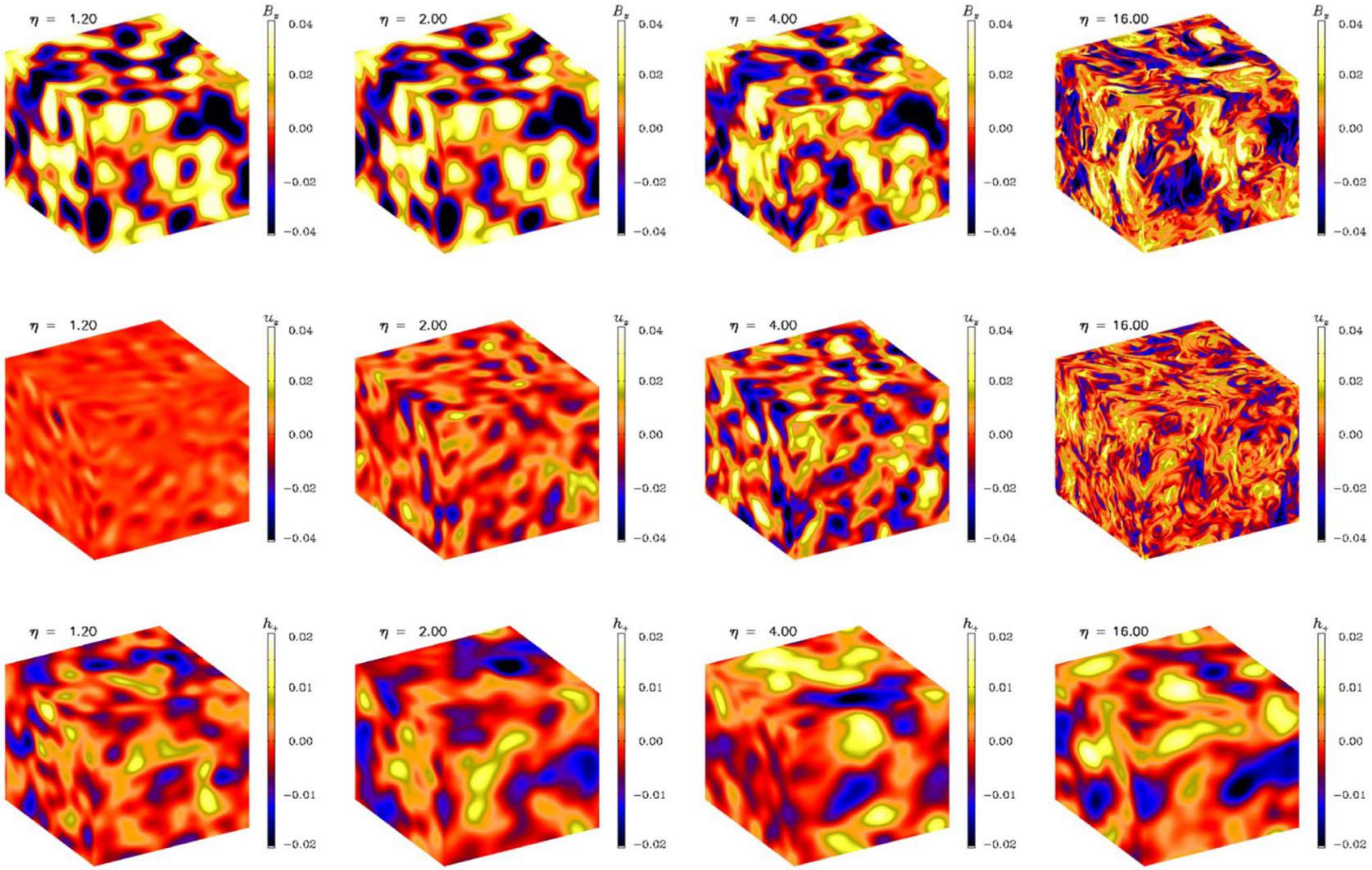
$$\begin{pmatrix} h \\ g \end{pmatrix}_{\text{2nd order}} = \ldots + \frac{\delta S}{k^2} \begin{pmatrix} [1-(\sin k\delta t)/k\delta t] \\ (1-\cos k\delta t)/\delta t \end{pmatrix}$$

→ Error decreases quadratically with decreasing time step dt

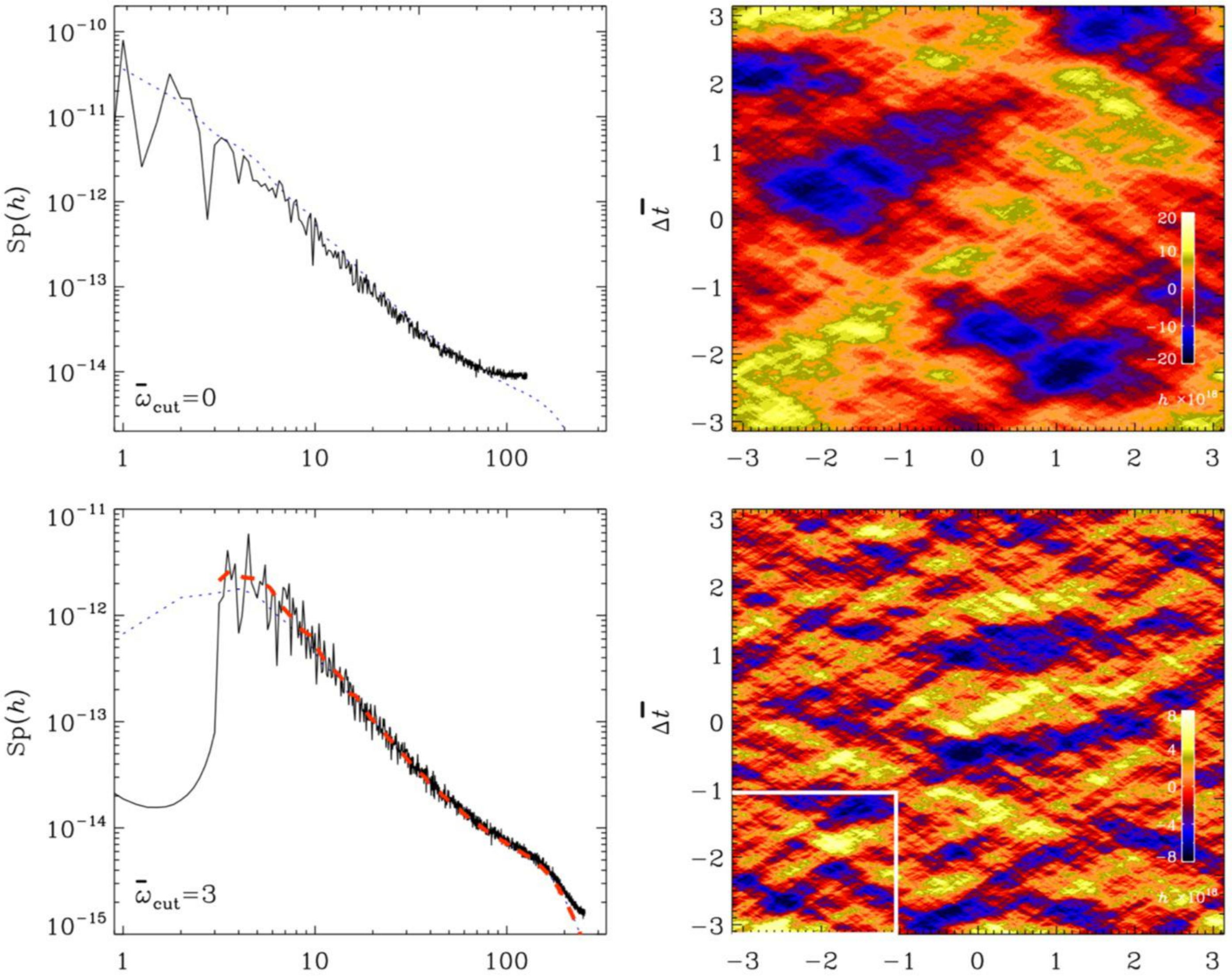
→ At no additional cost



No small scales in GW field



Temporal spectra and real space GW field



He, AB, Sinha (2021)

Here finite graviton mass

$$(\Box - m_{\rm g}^2)\bar{h}_{\mu\nu} = -16\pi G T_{\mu\nu},$$

Lower cutoff frequency

$$\omega_{\rm cut} = m_{\rm g} c^2/\hbar$$

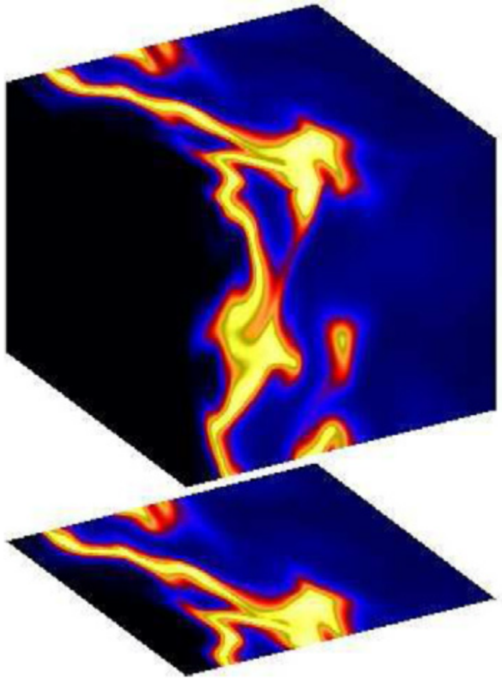
Cutoff dominates visual appearence

Again mostly large scales

A high-order public domain code for direct numerical simulations of turbulent combustion

N. Babkovskaia [a,*], N.E.L. Haugen [b], A. Brandenburg [c,d]

$$Y_{H_2} = 2.4\%,\ Y_{O_2} = 23\% \text{ and } Y_{N_2} = 74.6\%$$



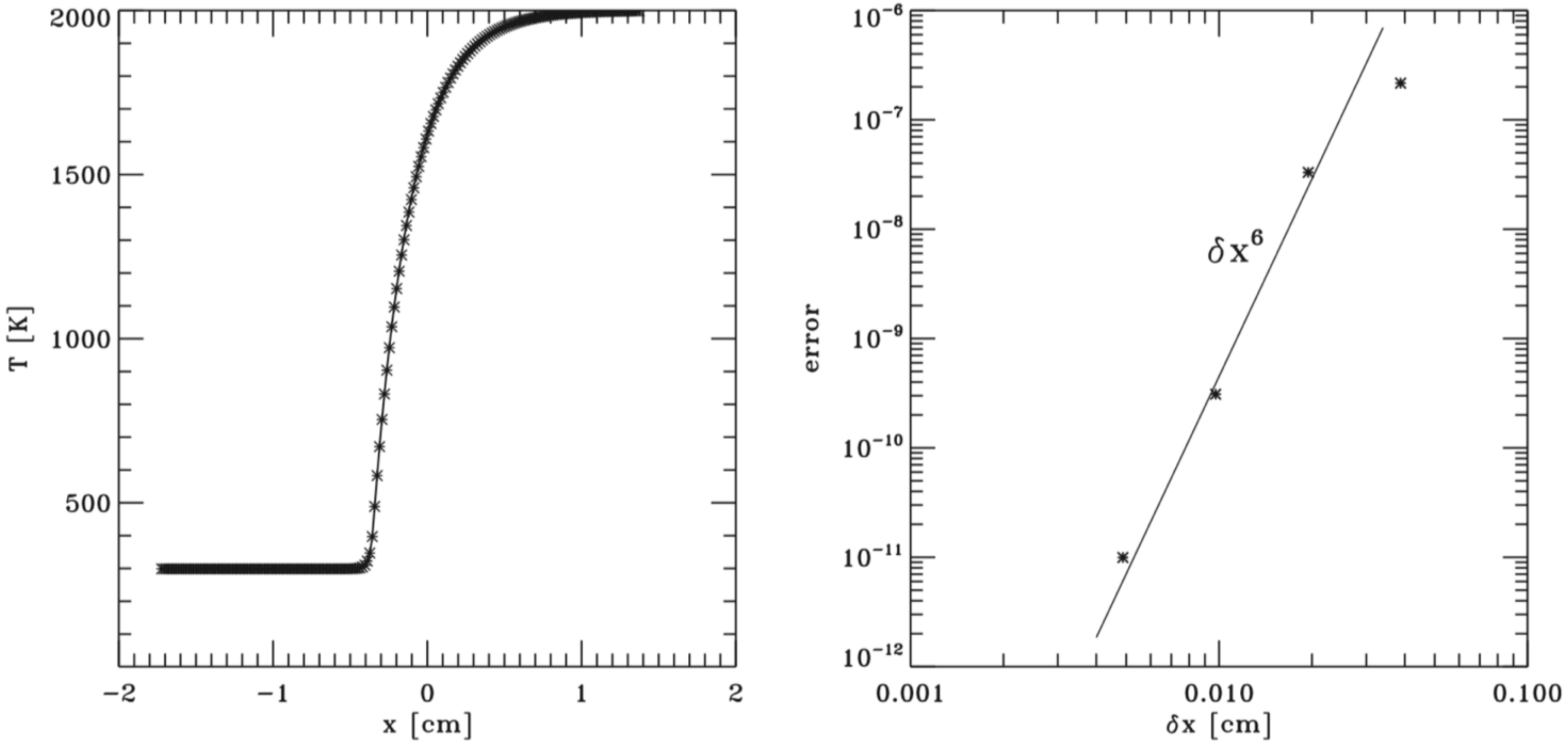


Fig. 1. One-step laminar premixed flame model. *Left panel*: temperature as a function of *x* obtained numerically (solid curve) and analytically (asterisks). *Right panel*: error of the calculation as a function of the mesh spacing δx is shown by asterisks, and the expected dependence of error (proportional to δx^6) is indicated by the solid line.